1. Consider a QFT of a Dirac spinor field $\Psi(x)$ and a massive vector field $A^\mu(x)$ governed by the Lagrangian

$$\mathcal{L} \;=\; \overline{\Psi}(i\partial\!\!\!/ - M)\Psi \;-\; \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} \;+\; \tfrac{1}{2}m^2 A_\mu A^\mu \;+\; g\,\overline{\Psi}\gamma^5 A\!\!\!/\Psi. \tag{1}$$

   (a) What are the symmetries — continuous and discrete — of this theory? Please explain how each symmetry acts on both the fields and the particles of the theory.

   (b) Write down the Feynman rules of this theory.

   (c) Consider the elastic scattering of two fermions, $f + f \to f + f$. Draw the Feynman diagrams contributing to this process (at the leading order of the perturbation theory) and apply the Feynman rules.

   (d) Calculate the actual scattering amplitude $\mathcal{M}(f + f \to f + f)$ in the non-relativistic limit of the fermions (assume $m \ll M$) and use it to calculate the effective force between the fermions. Hint: This force is spin-dependent.

2. Now consider a a new QFT where a real scalar field $\Phi_1(x)$, a real pseudoscalar $\Phi_2(x)$ and a Dirac spinor $\Psi(x)$ are governed by the Lagrangian

$$\begin{aligned}\mathcal{L} \;=\; &\overline{\Psi}(i\partial\!\!\!/ - M)\Psi \;+\; \tfrac{1}{2}(\partial_\mu\Phi_1)^2 \;+\; \tfrac{1}{2}(\partial_\mu\Phi_2)^2 \;-\; \tfrac{1}{2}m^2(\Phi_1^2 + \Phi_2^2)\\ &-\; g\,\overline{\Psi}(\Phi_1 + i\gamma^5\Phi_2)\Psi \;-\; \tfrac{1}{4}\lambda(\Phi_1^2 + \Phi_2^2)^2.\end{aligned} \tag{2}$$

   (a) Write down the Feynman rules of this theory.

   (b) Assume $m > 2M$ and calculate (to the lowest non-trivial order of the perturbation theory) the decay rates of the scalar and the pseudoscalar particles into fermion + antifermion pairs.

   (c) Generally, $\Gamma_S \neq \Gamma_P$, but the two decay rates become equal in the limit of negligible fermionic mass M. This equality follows from the axial symmetry of the Lagrangian (2) for $M = 0$. Write down the explicit action of this symmetry on the fermionic and bosonic fields of the theory and explain why it leads to $\Gamma_S = \Gamma_P$.

   (d) Finally, consider the *second*-lowest (*i.e.*, first sub-leading) order of the perturbation theory. Draw Feynman diagrams contributing to the decay amplitudes $\mathcal{M}(S \to f + af)$ and $\mathcal{M}(P \to f + af)$ at this order. Apply the Feynman rules but skip the actual calculations.