1. Consider an $SO(N)$ symmetric theory of N real scalar fields,

$$\mathcal{L} \;=\; \tfrac{1}{2}\partial_\mu\Phi^a\partial^\mu\Phi^a \;-\; \tfrac{1}{2}m^2\,\Phi^a\Phi^a \;-\; \tfrac{1}{4}\lambda\,(\Phi^a\Phi^a)^2\,. \tag{1}$$

   (a) Write down the Noether currents J^μ_{ab} of the $SO(N)$ symmetry, write down the classical field equations and verify that the symmetry currents are indeed conserved.

   (b) For the quantum theory, write the $SO(N)$ 'charges' $\hat{Q}_{ab} = \int d^3\mathbf{x}\,\hat{J}^0_{ab}$ in terms of bosonic creation and annihilation operators.

   Note: Expand the quantum fields $\hat{\Phi}^a(x)$ into creation and annihilation operators as if the fields were free, *i.e.* $\lambda = 0$.

   (c) Verify that the 'charges' $\hat{Q}_{ab}$ have correct commutation relations of the generators of the $SO(N)$ symmetry (which is isomorphic to the rotation symmetry in N Euclidean dimensions).

   Now let $m^2 < 0$. In the semiclassical limit, the potential $\frac{1}{2}m^2\Phi^a\Phi^a + \frac{1}{4}\lambda(\Phi^a\Phi^a)^2$ has a minimum at $\Phi^a\Phi^a = (-m^2/\lambda) > 0$ hence the fields develop symmetry-breaking vacuum expectation values; without loss of generality, we may assume $\langle\Phi^a\rangle = \delta_{a,1}\sqrt{-m^2/\lambda}$. The $SO(N)$ symmetry is now *spontaneously* broken down to its $SO(N-1)$ subgroup generated by the Q_{ab} for $a, b \neq 1$.

   (d) Derive the spectrum of this theory (semiclassical limit only) and verify that the massless particles are indeed Goldstone bosons of the broken symmetry, that is the $\hat{J}^\mu_{1a}(x)$ operator acting on the vacuum state creates a Goldstone particle of species a.

2. Consider a free non-hermitian (complex) quantum scalar field $\hat{\Phi}(x)$. Calculate the correlation function $\langle 0|\,T\left(\left(\hat{\Phi}(x)\right)^n\left(\hat{\Phi}^\dagger(y)\right)^n\right)|0\rangle$.

3. Derive the so-called *Gordon identity* for the Dirac spinors:

$$\bar{u}(p')\gamma^\mu u(p) \;=\; \frac{1}{2M}\,(p'+p)^\mu\,\bar{u}(p')u(p) \;+\; \frac{i}{M}\,(p'-p)^\nu\,\bar{u}(p')S^{\mu\nu}u(p)\,. \tag{2}$$

   Hint: Use the Dirac equation.