1. First, consider the Yukawa theory of a Dirac field Ψ and a real *pseudo*scalar field Φ,

$$\mathcal{L} \;=\; \tfrac{1}{2}\big(\partial_\mu\Phi\big)^2 \;-\; \tfrac{1}{2}m^2\Phi^2 \;+\; \overline{\Psi}(i\,\partial\!\!\!/ - M)\Psi \;-\; ig\Phi\times\overline{\Psi}\gamma^5\Psi \;-\; \tfrac{1}{24}\lambda\Phi^4.$$

At the tree level of the perturbation theory, calculate the annihilation cross section fermion + antifermion → 2 bosons. For simplicity, assume $m \ll M$.

2. Next, consider a rather complicated QFT comprising a charged Dirac field $\Psi_c(x)$, a neutral Dirac field $\Psi_n(x)$, a charged scalar field $\Phi(x)$ and the electromagnetic field $A^\mu(x)$. Classically — and at the tree level of the quantum theory,

$$\begin{aligned}\mathcal{L} \;=\; &-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} \;+\; D_\mu\Phi^* D^\mu\Phi \;-\; m_s^2\Phi^*\Phi \;+\; \overline{\Psi}_c(iD\!\!\!\!/ - m_c)\Psi_c \;+\; \overline{\Psi}_n(i\,\partial\!\!\!/ - m_n)\Psi_n\\ &+\; g\Phi\overline{\Psi}_c\Psi_n \;+\; g\Phi^*\overline{\Psi}_n\Psi_c \;-\; \tfrac{1}{4}\lambda\big(\Phi^*\Phi\big)^2.\end{aligned} \tag{1}$$

Let us focus on the electromagnetic form factors $F_1(q^2)$ and $F_2(q^2)$ of the *neutral* Dirac field Ψ_n.

(a) Draw *all* Feynman diagrams contributing to these form factors at the one-loop level of the perturbation theory. Do any of the counterterms of the renormalized theory contribute at this level? Explain your answer.

(b) Evaluate the diagrams and write them down as integrals over the Feynman parameters. Allow for generic masses m_n, m_c and m_s and any q^2.

(c) Verify that $F_1(q^2=0)=0$ and hence the radiative corrections do not 'endow' the Ψ_n field with a *net* electric charge. There is however a non-trivial electric charge density, as witnessed by $F_1(q^2)\neq 0$ for $q^2<0$.

(d) Finally, assume $m_n \approx m_c \gg m_s$ and calculate the magnetic dipole moment of the neutral fermion.

3. Now consider the Scalar QED as a renormalized quantum field theory.

(a) Write down all the counterterms needed to renormalize the Scalar QED. For each counterterm, write down the renormalization condition for its finite part in terms of some amplitude and draw *all* the tree and one-loop Feynman diagrams contributing to such amplitude as well as the counterterm.

(b) Now comes the hard part: Calculate the infinite parts of all the counterterms (to the one-loop order).

Actually, the δ_3 counterterm is already calculated in homework #17. Do not waste your time copying the posted solution; calculating all the other counterterms is work enough.

Note that some of the counterterms depend on the gauge-fixing parameter ξ of photonic propagators. You are free to choose any gauge you like, but make sure you use the same gauge gauge in all calculations. In my opinion, the Feynman gauge ($\xi = 1$) makes for simpler calculations, but many people prefer the Landau gauge ($\xi = 0$).

(c) Some counterterms are related by gauge invariance of the theory, *i.e.* $\delta_1 = \delta_2$ in fermionic QED. Write down similar Ward identities for the scalar QED and verify that they are indeed satisfied at the one-loop level of the perturbation theory.

Note: using different gauges to calculate different counterterms would break the Ward identities.

⋆ *For extra credit,* calculate the finite parts of the relevant counterterms and verify that they also satisfy the Ward identities.