

1. In the linear sigma model (*cf.* previous homework), calculate the tree-level decay rate $\Gamma(\sigma \rightarrow \pi\pi)$.
2. Consider two Dirac fields $\Psi_1(x)$ and $\Psi_2(x)$ coupled to the a real scalar field $\Phi(x)$:

$$\begin{aligned} \mathcal{L} = & \bar{\Psi}_1(i \not{\partial} - m_1)\Psi_1 + \bar{\Psi}_2(i \not{\partial} - m_2)\Psi_2 + \frac{1}{2}(\partial_\mu\Phi)^2 - \frac{1}{2}M^2\Phi^2 \\ & - g_1\Phi\bar{\Psi}_1\Psi_1 - g_2\Phi\bar{\Psi}_2\Psi_2. \end{aligned} \quad (1)$$

At the tree level, calculate the matrix element, the partial cross-section and the total cross-section for scattering of one type of a fermion off the other type, $f_1 + f_2 \rightarrow f_1 + f_2$. Take the initial fermions to be unpolarized and sum over the final fermion's polarizations.

Hint: Prove and use

$$\begin{aligned} \frac{1}{2} \sum_{s,s'} |\bar{u}(p', s')u(p, s)|^2 &= \frac{1}{2} \text{tr}((m + \not{p}')(m + \not{p})) \\ &= 2(m^2 + EE' - \mathbf{p}\mathbf{p}'). \end{aligned} \quad (2)$$

3. Consider Rutherford scattering of an electron off a static point-like electric charge. See problem 4.4 of the *Peskin & Schroeder* textbook for details and follow steps (a), (b) and (c) therein.
- (d) In addition, calculate the scattering cross-section for a relativistic unpolarized electron.

Hint: Prove and use

$$\begin{aligned} \frac{1}{2} \sum_{s,s'} |\bar{u}(p', s')\gamma^0 u(p, s)|^2 &= \frac{1}{2} \text{tr}((m + \not{p}')\gamma^0(m + \not{p})\gamma^0) \\ &= 2(m^2 + EE' + \mathbf{p}\mathbf{p}'). \end{aligned} \quad (3)$$