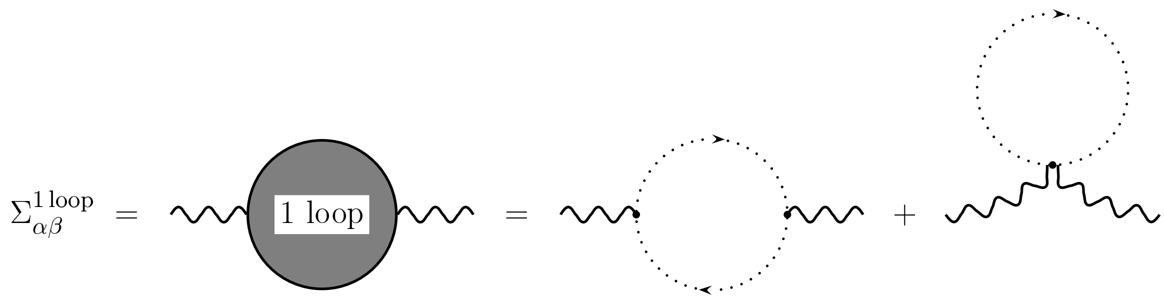
1. The first problem concerns electric charge renormalization in the *scalar QED* — the theory of the EM field $A^\mu(x)$ coupled to a *charged* scalar field $\Phi(x)$. At the one-loop level, there are two Feynman diagrams contributing to the 1PI two-photon amplitude, namely



   (a) Evaluate the two diagrams using dimensional regularization and verify that

$$\Sigma^{1\,\text{loop}}_{\alpha\beta}(k) \;=\; \left(-g_{\alpha\beta}k^2 + k_\alpha k_\beta\right) \times \mathcal{B}^{1\,\text{loop}}(k^2) \tag{1}$$

   (b) Calculate $\mathcal{B}^{1\,\text{loop}}(k^2)$ and use it to relate the physical coupling $\alpha = e^2/4\pi$ and the bare coupling $\alpha_0 = e_0^2/4\pi$. Also, calculate the effective coupling at high energies (*i.e.*, short distances) and show that

$$\frac{1}{\alpha_{\text{eff}}(k^2)} \;\approx\; \frac{1}{\alpha(0)} \;-\; \frac{1}{12\pi}\left(\log\frac{-k^2}{m^2} \;-\; \frac{8}{3}\right). \tag{2}$$

2. The second problem is a reading assignment. Read §7.3 of the *Peskin & Schroeder* textbook and learn about the Optical Theorem in Quantum field theory and its implications for imaginary parts of Feynman diagrams. Also, read the related discussion in §7.5 of the imaginary part of $\mathcal{B}^{1\,\text{loop}}$ (called $\hat{\Pi}_2$ in the textbook) for $k^2 > +4m_e^2$.