In the center-of-mass frame, the partial cross-section for a two particles → two particles process $A+B\to 1+2$ is calculated in the Peskin & Schroeder textbook in eq. (4.84):

$$\left(\frac{d\sigma}{d\Omega}\right)_{\rm c.m.} = \frac{1}{2E_A\,2E_b\,|v_A-v_B|}\times\frac{|\mathbf{p}_1|}{(2\pi)^2 4E_{\rm c.m.}}\times|\mathcal{M}(A+B\to 1+2)|^2 \qquad (4.84)$$

$$= \frac{|\mathbf{p}_1|}{|\mathbf{p}_A|}\times\frac{|\mathcal{M}|^2}{64\pi^2 E^2_{\rm c.m.}}\,. \qquad (\text{S.1})$$

When one or both of the final-state particles have spin but its polarization is not measured in the experiment, one should sum the $|\mathcal{M}|^2$ over the final particles' spins. Likewise, when the initial particles have spins but the colliding beams are un-polarized — that is, the initial particles are equally likely to have any particular spin state — one should average the $|\mathcal{M}|^2$ over all such spin states. Thus, for the $e^++e^-\to H^0+Z^0$ process at hand, we have

$$\left(\frac{d\sigma}{d\Omega}\right)_{\rm c.m.} = \frac{|\mathbf{p}_Z|}{|\mathbf{p}_e|}\times\frac{1}{64\pi^2 E^2_{\rm c.m.}}\times\left(\overline{|\mathcal{M}|^2}\stackrel{\rm def}{=}\frac{1}{2}\sum_{s(e^+)}\frac{1}{2}\sum_{s(e^-)}\sum_{s(Z^0)}|\mathcal{M}|^2\right), \qquad (\text{S.2})$$

and according to eq. (4) of the problem set,

$$\overline{|\mathcal{M}(e^++e^-\to H^0+Z^0)|^2} = A\left(\frac{e^2M_{Z^0}}{E^2_{\rm c.m.}-M^2_{Z^0}}\right)^2\times\left((p_{e^+}\cdot p_{e^-}) + \frac{2}{M^2_{Z^0}}(p_{Z^0}\cdot p_{e^+})(p_{Z^0}\cdot p_{e^-})\right). \qquad (4)$$

In terms of the center-of-mass-frame energies and momenta, $\mathbf{p}_{e^+}=-\mathbf{p}_{e^-}$, $E_{e^+}=E_{e^-}=\frac{1}{2}E_{\rm c.m.}$, thus

$$(p_{e^+}\cdot p_{e^-}) = E^2_{e^-} + \mathbf{p}^2_{e^-} \approx 2E^2_{e^-} = \tfrac{1}{2}E^2_{\rm c.m.} \qquad (\text{S.3})$$

Also,

$$(p_{Z^0}\cdot p_{e^\pm}) = E_{Z^0}E_{e^\pm} - \mathbf{p}_{Z^0}\cdot\mathbf{p}_{e^\pm} = \tfrac{1}{2}E_{\rm c.m.}\,(E_{Z^0}\pm|\mathbf{p}_{Z^0}|\cos\theta) \qquad (\text{S.4})$$

where θ is the angle between the directions of the initial electron and the final Z^0 particle — or equivalently, between the initial positron and the final Higgs particle — and consequently

$$(p_{Z^0}\cdot p_{e^+})(p_{Z^0}\cdot p_{e^-}) \;=\; \tfrac{1}{4}E_{\text{c.m.}}^2\left(E_{Z^0}^2-\mathbf{p}_{Z^0}^2\cos^2\theta\right) = \tfrac{1}{4}E_{\text{c.m.}}^2\left(E_{Z^0}^2\sin^2\theta+M_{Z^0}^2\cos^2\theta\right). \tag{S.5}$$

Combining eqs. (4), (S.3) and (S.5) together, we have

$$\overline{|\mathcal{M}(e^++e^-\to H^0+Z^0)|^2} \;=\; \frac{\frac{1}{2}Ae^4\,E_{\text{c.m.}}^2M_{Z^0}^2}{(E_{\text{c.m.}}^2-M_{Z^0}^2)^2}\times\left(2\;+\;\frac{\mathbf{p}_{Z^0}^2}{M_{Z^0}^2}\sin^2\theta\right) \tag{S.6}$$

and consequently,

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{c.m.}} \;=\; \frac{|\mathbf{p}_{Z^0}|}{|\mathbf{p}_e|}\times\frac{A\alpha_{EM}^2M_{Z^0}^2}{8(E_{\text{c.m.}}^2-M_{Z^0}^2)^2}\times\left(2\;+\;\frac{\mathbf{p}_{Z^0}^2}{M_{Z^0}^2}\sin^2\theta\right) \tag{S.7}$$

It remains to calculate the momenta of the Z^0 particle and the Higgs particle produced in the $e^+ + e^-$ collision. Actually, it is easier to calculate their energies first, from the following equations:

$$\begin{aligned} E_{Z^0}\;+\;E_H \;&=\; E_{\text{c.m.}}\,, \\ E_{Z^0}^2\;-\;E_H^2 \;&=\; (\mathbf{p}_{Z^0}^2+M_{Z^0}^2)\;-\;(\mathbf{p}_H^2+M_H^2) \;=\; M_{Z^0}^2\;-\;M_H^2\,, \\ E_{Z^0}\;-\;E_H \;&=\; \frac{E_{Z^0}^2-E_H^2}{E_{Z^0}+E_H} \;=\; \frac{M_{Z^0}^2-M_H^2}{E_{\text{c.m.}}}\,, \end{aligned} \tag{S.8}$$

and hence

$$E_{Z^0} \;=\; \frac{E_{\text{c.m.}}^2+M_{Z^0}^2-M_H^2}{2E_{\text{c.m.}}}\,,\qquad E_H \;=\; \frac{E_{\text{c.m.}}^2+M_H^2-M_{Z^0}^2}{2E_{\text{c.m.}}}\,. \tag{S.9}$$

Consequently,

$$\begin{aligned} \mathbf{p}_{Z^0}^2 \;=\; \mathbf{p}_H^2 \;&=\; E_{Z^0}^2\;-\;M_{Z^0}^2 \;=\; E_H^2\;-\;M_H^2 \\ &=\; \frac{1}{4E_{\text{c.m.}}^2}\left(E_{\text{c.m.}}^4\;-\;2E_{\text{c.m.}}^2(M_{Z^0}^2+M_H^2)\;+\;(M_{Z^0}^2-M_H^2)^2\right) \\ &=\; \frac{1}{4E_{\text{c.m.}}^2}\left(E_{\text{c.m.}}^2-(M_H+M_{Z^0})^2\right)\left(E_{\text{c.m.}}^2-(M_H-M_{Z^0})^2\right). \end{aligned} \tag{S.10}$$

At this point, we are finally ready to discuss the angular distribution of the Z^0 and Higgs particles produced in the electron-positron collision. This distribution is governed by the last factor on the right hand side of eq. (S.7) and its shape depends on the collision's energy. At energies just above the Z^0+Higgs thresholds, $E_{\text{c.m.}} = M_{Z^0} + M_H + \epsilon$ — which is are the highest energies available at the LEP II accelerator assuming the Higgs particle is as light as it can possibly be (in light of current experimental data) — the final-state Z^0 and Higgs particle have low (non-relativistic) momenta, $\mathbf{p}_{Z^0}^2 \ll M_{Z^0}^2$ (see eq. (S.10)), so their angular distribution is approximately isotropic. At higher collision energies — hopefully available at future accelerators — the Z^0 particle becomes relativistic and hence, according to eq. (S.7), it is more likely to fly away perpendicular to the $e^+ + e^-$ collision axis than along it.

Ultimately, in the extremely high energy limit we have $\mathbf{p}_{Z^0}^2 \approx (\frac{1}{2}E_{\text{c.m.}})^2 \gg M_{Z^0}^2$ and the partial cross section (S.7) is proportional to $\sin^2\theta$. In this regime, the angular distribution of the Z^0 particles — and hence the Higgs particles as well — is similar to the angular distribution of the EM radiation of an eclectic dipole. The reason for this similarity is that the Z^0 and the photon are both vector (spin=1) particles with similar couplings to electrons — and the ultra-relativistic Z^0 particles are in effect approximately massless.

Finally, consider the total cross-section σ of the $e^+ + e^- \to Z^0 + H$ process as a function of the collision energy. Integration eq. (S.7) over the solid angle $d\Omega$ and substituting eq. (S.10) for the $|\mathbf{p}_{Z^0}|$, we arrive at

$$\sigma(e^+ + e^- \to Z^0 + H) \;=\; \frac{\pi A\alpha_{EM}^2}{12} \times \frac{E_{\text{c.m.}}^2}{(E_{\text{c.m.}}^2 - M_{Z^0}^2)^2} \times \left(\beta^3 \,+\, \frac{12\beta M_{Z^0}^2}{E_{\text{c.m.}}^2}\right) \tag{S.11}$$

where

$$\beta \;\equiv\; \frac{|\mathbf{p}_{Z^0}|}{\frac{1}{2}E_{\text{c.m.}}} \;=\; \sqrt{1-\left(\frac{M_{Z^0}+M_H}{E_{\text{c.m.}}}\right)^2}\,\sqrt{1-\left(\frac{M_{Z^0}-M_H}{E_{\text{c.m.}}}\right)^2}\,. \tag{S.12}$$

Near the energy threshold, $E_{\text{c.m.}} = M_{Z^0} + M_H + \epsilon$, the cross-section

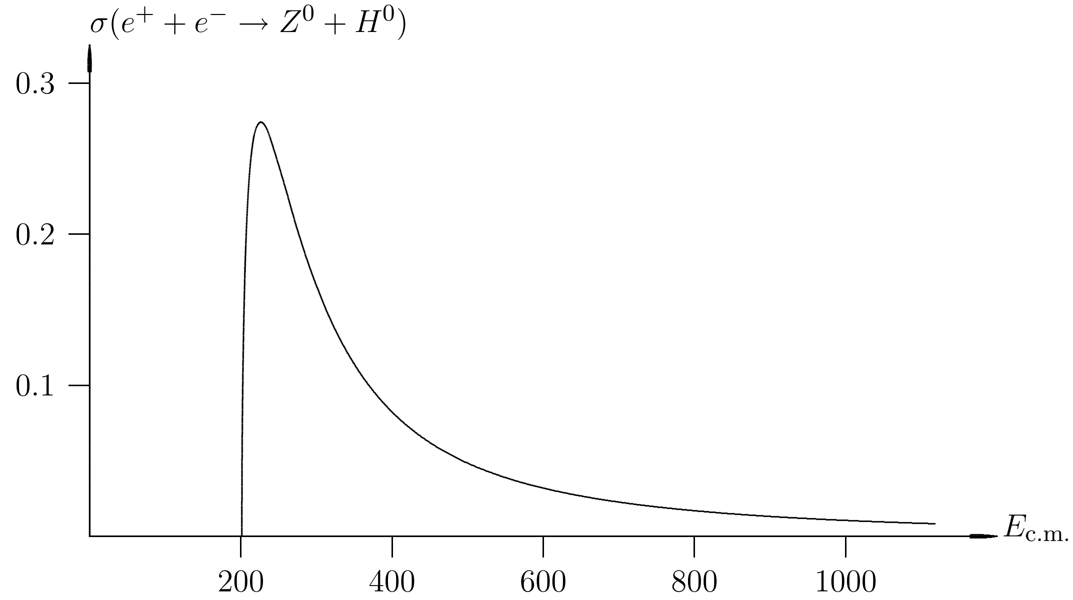
$$\sigma(e^+ + e^- \to Z^0 + H) \;\approx\; \frac{2\pi\sqrt{2}A\alpha_{EM}^2\, M_{Z^0}^{5/2}\,\epsilon^{1/2}}{M_N^{3/2}(M_H + M_{Z^0})^{3/2}(M_H + 2M_{Z^0})^2} \tag{S.13}$$

raises with energy as $\sqrt{\epsilon}$, but not too far above the threshold, the cross-section reaches its maximal value and starts decreasing. At very high energies, we have

$$\sigma(e^+ + e^- \to Z^0 + H) \;\approx\; \frac{\pi A\alpha_{EM}^2}{12E_{\text{c.m.}}^2} \tag{S.14}$$

and the cross-section decreases with the energy as $1/E_{\text{c.m.}}^2$.

For generic energies, eq. (S.11) is too complicated to study analytically. Instead, let me simply show you the numeric plot:



On this plot, $E_{\text{c.m.}}$ is in GeV, the cross-section σ is in picobarns (1 pb $= 10^{-36}$ cm^2) and the Higgs's mass is assumed to be $M_H = 110$ GeV, which is a bit heavier than the $M_{Z^0} = 91.4$ GeV.