

First, a reading assignment: *Carefully* study section §4.5 of the *Peskin & Schroeder* textbook (same as last homework), and this time pay attention to the phase-space factors of decay and scattering processes when the final state has more than two particles.

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And as a practical application of this reading assignment, consider the muon decay,  $\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$ . Since neutrinos are hard to detect experimentally, the readily measurable quantities for this process are the total muon decay rate  $\Gamma_\mu = 1/\tau_\mu$  and the energy distribution of electrons produced by decaying muons; the latter is known to have a maximum at the highest kinematically allowed value of  $E_e$ .

According to the Fermi theory of weak interactions, the matrix element for muon decay is

$$\langle e^-, \bar{\nu}_e, \nu_\mu | \mathcal{M} | \mu^- \rangle = \frac{G_F}{\sqrt{2}} [\bar{u}(\nu_\mu)(1 - \gamma^5)\gamma^\alpha u(\mu^-)] \times [\bar{u}(e^-)(1 - \gamma^5)\gamma_\alpha v(\bar{\nu}_e)]. \quad (1)$$

The modern Standard Model of particle interactions produces essentially the same answer at the tree level of the perturbation theory.

Later in this class we shall learn how to sum  $|\mathcal{M}|^2$  over the fermionic spins. In particular, in a future homework you will show that

$$\frac{1}{2} \sum_{\text{all spins}} |\langle e^-, \bar{\nu}_e, \nu_\mu | \mathcal{M} | \mu^- \rangle|^2 = 64 G_F^2 (p_\mu \cdot p_{\bar{\nu}}) (p_e \cdot p_\nu). \quad (2)$$

But in this homework, you should simply take this formula and use it to calculate the  $d\Gamma/dE_e$  and the  $\Gamma_{\text{tot}}$  in the muon's rest frame. This is a straightforward but non-trivial exercise because of three particles in the final state, with momenta subject to constraints

$$\mathbf{p}_e + \mathbf{p}_\nu + \mathbf{p}_{\bar{\nu}} = \mathbf{0}, \quad E_e + E_\nu + E_{\bar{\nu}} = M_\mu \approx 105.66 \text{ MeV}. \quad (3)$$

Fortunately, the neutrinos are massless while the electron may be approximated as massless because in most decay events the electron's energy  $E_e = O(M_\mu) \gg m_e$ . You are advised to take this approximation  $m_e \approx 0$  as it simplifies the calculation quite a bit.