In the center-of-mass frame, the partial cross-section for a two particles → two particles process $A + B \to 1 + 2$ is calculated in the Peskin & Schroeder textbook in eq. (4.84):

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{c.m.}} = \frac{1}{2E_A\,2E_b\,|v_A - v_B|} \times \frac{|\mathbf{p}_1|}{(2\pi)^2 4E_{\text{c.m.}}} \times |\mathcal{M}(A+B\to 1+2)|^2 \tag{4.84}$$

$$= \frac{|\mathbf{p}_1|}{|\mathbf{p}_A|} \times \frac{|\mathcal{M}|^2}{64\pi^2 E_{\text{c.m.}}^2}\,. \tag{S.1}$$

When one or both of the final-state particles have spin but its polarization is not measured in the experiment, one should sum the $|\mathcal{M}|^2$ over the final particles' spins. Likewise, when the initial particles have spins but the colliding beams are un-polarized — that is, the initial particles are equally likely to have any particular spin state — one should average the $|\mathcal{M}|^2$ over all such spin states. Thus, for the $e^+ + e^- \to H^0 + Z^0$ process at hand, we have

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{c.m.}} = \frac{|\mathbf{p}_Z|}{|\mathbf{p}_e|} \times \frac{1}{64\pi^2 E_{\text{c.m.}}^2} \times \left(\overline{|\mathcal{M}|^2} \stackrel{\text{def}}{=} \tfrac{1}{2}\sum_{s(e^+)} \tfrac{1}{2}\sum_{s(e^-)} \sum_{s(Z^0)} |\mathcal{M}|^2\right), \tag{S.2}$$

and according to eq. (4) of the problem set,

$$\overline{|\mathcal{M}(e^+ + e^- \to H^0 + Z^0)|^2} = A\left(\frac{e^2 M_{Z^0}}{E_{\text{c.m.}}^2 - M_{Z^0}^2}\right)^2 \times \left((p_{e^+}\cdot p_{e^-}) + \frac{2}{M_{Z^0}^2}(p_{Z^0}\cdot p_{e^+})(p_{Z^0}\cdot p_{e^-})\right). \tag{4}$$

In terms of the center-of-mass-frame energies and momenta, $\mathbf{p}_{e^+} = -\mathbf{p}_{e^-}$, $E_{e^+} = E_{e^-} = \frac{1}{2}E_{\text{c.m.}}$, thus

$$(p_{e^+}\cdot p_{e^-}) = E_{e^-}^2 + \mathbf{p}_{e^-}^2 \approx 2E_{e^-}^2 = \tfrac{1}{2}E_{\text{c.m.}}^2 \tag{S.3}$$

Also,

$$(p_{Z^0}\cdot p_{e^\pm}) = E_{Z^0}E_{e^\pm} - \mathbf{p}_{Z^0}\cdot\mathbf{p}_{e^\pm} = \tfrac{1}{2}E_{\text{c.m.}}\,(E_{Z^0} \pm |\mathbf{p}_{Z^0}|\cos\theta) \tag{S.4}$$

where θ is the angle between the directions of the initial electron and the final Z^0 particle — or equivalently, between the initial positron and the final Higgs particle — and consequently

$$(p_{Z^0}\cdot p_{e^+})(p_{Z^0}\cdot p_{e^-}) \;=\; \tfrac{1}{4}E^2_{\rm c.m.}\left(E^2_{Z^0}-\mathbf{p}^2_{Z^0}\cos^2\theta\right) =\; \tfrac{1}{4}E^2_{\rm c.m.}\left(E^2_{Z^0}\sin^2\theta+M^2_{Z^0}\cos^2\theta\right). \qquad \text{(S.5)}$$

Combining eqs. (4), (S.3) and (S.5) together, we have

$$\overline{\left|\mathcal{M}(e^+ + e^- \to H^0 + Z^0)\right|^2} \;=\; \frac{\frac{1}{2}Ae^4\,E^2_{\rm c.m.}M^2_{Z^0}}{(E^2_{\rm c.m.}-M^2_{Z^0})^2}\times\left(2\;+\;\frac{\mathbf{p}^2_{Z^0}}{M^2_{Z^0}}\,\sin^2\theta\right) \qquad \text{(S.6)}$$

and consequently,

$$\left(\frac{d\sigma}{d\Omega}\right)_{\rm c.m.} \;=\; \frac{|\mathbf{p}_{Z^0}|}{|\mathbf{p}_e|}\times\frac{A\alpha^2_{EM}M^2_{Z^0}}{8(E^2_{\rm c.m.}-M^2_{Z^0})^2}\times\left(2\;+\;\frac{\mathbf{p}^2_{Z^0}}{M^2_{Z^0}}\,\sin^2\theta\right) \qquad \text{(S.7)}$$

The last factor here governs the angular distribution of the Z^0 and Higgs particles produced in the electron-positron collision. The exact shape of this distribution depends on the collision's energy via the $\mathbf{p}^2_{Z_0}/M^2_{Z_0}$ coefficient, so our next task is to calculate the $\mathbf{p}^2_{Z_0}=\mathbf{p}^2_H$.

The energies of the Z^0 particle and the Higgs particle produced in the $e^+ + e^-$ collision obey the following equations:

$$\begin{aligned}
E_{Z^0} \;+\; E_H \;&=\; E_{\rm c.m.}\,,\\
E^2_{Z^0} \;-\; E^2_H \;&=\; (\mathbf{p}^2_{Z^0}+M^2_{Z^0}) \;-\; (\mathbf{p}^2_H+M^2_H) \;=\; M^2_{Z^0} \;-\; M^2_H\,,\\
E_{Z^0} \;-\; E_H \;&=\; \frac{E^2_{Z^0}-E^2_H}{E_{Z^0}+E_H} \;=\; \frac{M^2_{Z^0}-M^2_H}{E_{\rm c.m.}}\,,
\end{aligned} \qquad \text{(S.8)}$$

and hence

$$E_{Z^0} \;=\; \frac{E^2_{\rm c.m.}+M^2_{Z^0}-M^2_H}{2E_{\rm c.m.}}\,, \qquad E_H \;=\; \frac{E^2_{\rm c.m.}+M^2_H-M^2_{Z^0}}{2E_{\rm c.m.}}\,. \qquad \text{(S.9)}$$

Consequently,

$$\begin{aligned}
\mathbf{p}^2_{Z^0} \;=\; \mathbf{p}^2_H \;&=\; E^2_{Z^0} \;-\; M^2_{Z^0} \;=\; E^2_H \;-\; M^2_H\\
&=\; \frac{1}{4E^2_{\rm c.m.}}\left(E^4_{\rm c.m.} \;-\; 2E^2_{\rm c.m.}(M^2_{Z^0}+M^2_H) \;+\; (M^2_{Z^0}-M^2_H)^2\right)\\
&=\; \frac{1}{4E^2_{\rm c.m.}}\left(E^2_{\rm c.m.}-(M_H+M_{Z^0})^2\right)\left(E^2_{\rm c.m.}-(M_H-M_{Z^0})^2\right).
\end{aligned} \qquad \text{(S.10)}$$

The highest total energies available at the LEP II accelerator were barely above the Z^0+Higgs threshold — and that's assuming the Higgs is as light as it could possibly be in light of current experimental data. In other words,

$$E_{\text{c.m.}} \;=\; E_{Z_0} \;+\; E_H \;=\; M_{Z_0} \;+\; M_H \;+\; \epsilon \tag{S.11}$$

for a rather small ϵ (energy above the threshold), which means that the final-state particles must have low (non-relativistic) momenta. Indeed according to eq. (S.10), in this regime $\mathbf{p}_{Z_0}^2 \approx 2\epsilon M_{Z_0} M_H/(M_{Z_0} + M_H) \ll M_{Z_0}^2$, and hence according to eq. (S.7) the angular distribution of the final-state particles is approximately isotropic.

At higher collision energies — hopefully available at future accelerators — the Z^0 particle would be relativistic and hence more likely to fly away perpendicular to the $e^+ + e^-$ collision axis than along it. Ultimately, in the extremely high energy limit we have $\mathbf{p}_{Z^0}^2 \approx (\frac{1}{2}E_{\text{c.m.}})^2 \gg M_{Z^0}^2$ and the partial cross section (S.7) is proportional to $\sin^2\theta$. In this regime, the angular distribution of the Z^0 particles — and hence the Higgs particles as well — is similar to the angular distribution of the EM radiation of an eclectic dipole. The reason for this similarity is that the Z^0 and the photon are both vector (spin=1) particles with similar couplings to electrons — and the ultra-relativistic Z^0 particles are in effect approximately massless.

Finally, consider the total cross-section σ of the $e^+ + e^- \to Z^0 + H$ process as a function of the collision energy. Integration eq. (S.7) over the solid angle $d\Omega$ and substituting eq. (S.10) for the $|\mathbf{p}_{Z^0}|$, we arrive at

$$\sigma_{\text{tot}}(e^+ + e^- \to Z^0 + H) \;=\; \frac{\pi A \alpha_{EM}^2}{12} \times \frac{E_{\text{c.m.}}^2}{(E_{\text{c.m.}}^2 - M_{Z^0}^2)^2} \times \left(\beta^3 \;+\; \frac{12\beta M_{Z^0}^2}{E_{\text{c.m.}}^2}\right) \tag{S.12}$$

where

$$\beta \;\equiv\; \frac{|\mathbf{p}_{Z^0}|}{\frac{1}{2}E_{\text{c.m.}}} \;=\; \sqrt{1 - \left(\frac{M_{Z^0} + M_H}{E_{\text{c.m.}}}\right)^2}\,\sqrt{1 - \left(\frac{M_{Z^0} - M_H}{E_{\text{c.m.}}}\right)^2}\,. \tag{S.13}$$

Near the energy threshold (S.11), the cross-section

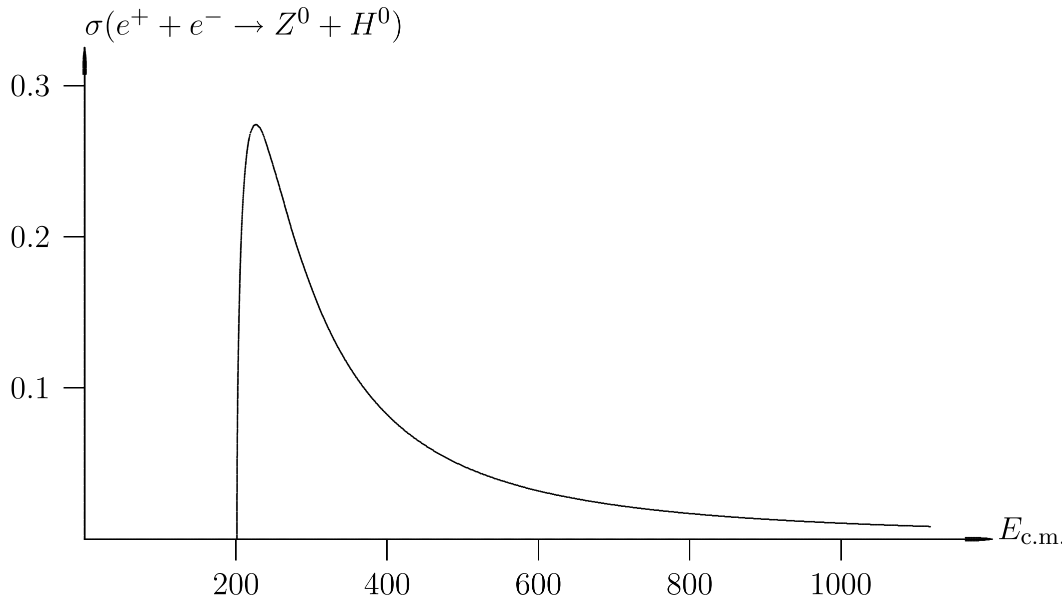
$$\sigma_{\text{tot}}(e^+ + e^- \to Z^0 + H) \;\approx\; \frac{2\pi\sqrt{2}A\alpha_{EM}^2\, M_{Z^0}^{5/2}\,\epsilon^{1/2}}{M_N^{3/2}(M_H + M_{Z^0})^{3/2}(M_H + 2M_{Z^0})^2} \tag{S.14}$$

raises with energy as $\sqrt{\epsilon}$, but not too far above the threshold σ_{tot} reaches its maximal value and starts decreasing. At very high energies, we have

$$\sigma(e^+ + e^- \to Z^0 + H) \;\approx\; \frac{\pi A\alpha_{EM}^2}{12E_{\text{c.m.}}^2} \tag{S.15}$$

and the cross-section decreases with the energy as $1/E_{\text{c.m.}}^2$.

For generic energies, eq. (S.12) is too complicated to study analytically. Instead, let me simply show you the numeric plot:



On this plot, $E_{\text{c.m.}}$ is in GeV, the cross-section σ is in picobarns (1 pb $= 10^{-36}$ cm^2) and the Higgs mass is assumed to be $M_H = 110$ GeV, which is a bit heavier than the $M_{Z^0} = 91.4$ GeV.