1. First, a reading assignment: §7.4 of the *Peskin & Schroeder* textbook about the diagrammatic proof of Ward–Takahashi identities for QED.

2. Next, consider "scalar QED", the theory of a charged scalar field $\phi(x) \neq \phi^*(x)$ coupled to the electromagnetic field $A^\mu(x)$,

$$\mathcal{L} \;=\; -\tfrac{1}{4}F_{\mu\nu}^2 \;+\; D^\mu\phi^* D_\mu\phi \;-\; M^2\phi^*\phi \;-\; \tfrac{1}{4}\lambda(\phi^*\phi)^2. \tag{1}$$

   Write down Ward–Takahashi identities for this theory and prove them using the diagrammatic method.

3. Finally, consider a theory comprised of the EM field $A^\mu(x)$, a charged scalar field $\phi(x)$, a charged Dirac field $\psi(x)$, and a neutral Dirac field $\chi(x)$, interacting with each other according to

$$\begin{aligned}\mathcal{L} \;=\; & -\tfrac{1}{4}F_{\mu\nu}^2 \;+\; D^\mu\phi^* D_\mu\phi \;-\; M^2\phi^*\phi \;-\; \tfrac{1}{4}\lambda(\phi^*\phi)^2 \\ & +\; \overline{\psi}(i\not{D} - m)\psi \;+\; \overline{\chi}(i\not{\partial} - m')\chi \;-\; \gamma\phi\overline{\psi}\chi \;-\; \gamma\phi^*\overline{\chi}\psi.\end{aligned} \tag{2}$$

   Write down the Ward–Takahashi identities for this theory and sketch out a proof.