Textbook problem 6.12:

(a) The impulse of a time-dependent force $F(t)$ is the integral

$$I = \int F(t) \, dt \quad \text{(S.1)}$$

Graphically, this integral is the area on the force-vs-time plot between the $t$ axis and the $F(t)$ line.

The $F(t)$ line on figure P6.12 comprises 3 straight-line segments, and the geometric figure enclosed between these segments and the $t$ axis is a trapeze. The area of a trapeze with parallel sides $a$ and $b$ and height (the distance between those parallel sides) $h$ is

$$A = \frac{a + b}{2} \times h. \quad \text{(S.2)}$$

For the trapeze on figure P6.12, $h$ is the maximal force $F_{\text{max}} = 4 \text{ N}$, $b$ is time interval $t_{\text{max}} = 1 \text{ s}$ between 2 s and 3 s during which the force takes its maximal value, and $a$ is the overall times interval $t_{\text{tot}} = 5 \text{ s}$ between 0 s and 5 s. Thus, the impulse is

$$I = \frac{t_{\text{tot}} + t_{\text{max}}}{2} \times F_{\text{max}} = \frac{5 \text{ s} + 1 \text{ s}}{2} \times 4 \text{ N} = 12 \text{ N} \cdot \text{s.} \quad \text{(S.3)}$$

Alternatively, we may treat the area under the $F(t)$ line as a sum of two right triangles and one rectangle. Both triangles have base $= 2 \text{ s}$ and height $= 4 \text{ N}$; the rectangle has base $= 1 \text{ s}$ and height $= 4 \text{ N}$. Thus, the total area is

$$A = 2 \times \frac{2 \text{ s} \times 4 \text{ N}}{2} + 1 \text{ s} \times 4 \text{ N} = 12 \text{ N} \cdot \text{s}, \quad \text{(S.4)}$$

and the impulse of the force depicted on figure P6.12 is 12 Newton-seconds.
(b) By the impulse-momentum theorem, the momentum of a body changes by the amount equal to net impulse of all the forces acting on it,

\[ \Delta P = I_{\text{net}}. \]  

(S.5)

In the problem at hand, there is only one force and its impulse is 12 N \cdot s = 12 \text{ kg} \cdot \text{m/s}, so the particle momentum changes by this amount.

If the particle is initially at rest, its initial momentum \( p_1 = mv_1 = 0 \), so after the force has acted, it acquires momentum \( p_2 = 12 \text{ kg} \cdot \text{m/s} \). Consequently, its velocity becomes

\[
v_2 = \frac{p_2}{m} = \frac{12 \text{ kg} \cdot \text{m/s}}{2 \text{ kg}} = 6 \text{ m/s}.
\]  

(S.6)

(c) This time we have a non-zero initial velocity, so according to the momentum-impulse theorem

\[
I = \Delta p \equiv p_2 - p_1 = mv_2 - mv_1.
\]  

(S.7)

solving this equation for the final velocity \( v_2 \) we find

\[
v_2 = v_1 + \frac{I}{m} = -2 \text{ m/s} + \frac{12 \text{ kg} \cdot \text{m/s}}{2 \text{ kg}} = +4 \text{ m/s}.
\]  

(S.8)

Textbook problem 6.14:
First, let’s determine the impulse of the force between the wall and the ball. By the impulse-momentum theorem,

\[
\vec{I} = \Delta \vec{p} \equiv \vec{p}_2 - \vec{p}_1 = m\vec{v}_2 - m\vec{v}_1,
\]  

(S.9)

and it’s important to remember that the impulse and the momenta are vectors. In components,

\[
I_x = mv_{2x} - mv_{1x} = mv_2 \cos \theta_2 - mv_1 \cos \theta_1,
\]

\[
I_y = mv_{2y} - mv_{1y} = mv_2 \sin \theta_2 - mv_1 \sin \theta_1,
\]  

(S.10)

where the angles \( \theta_1 \) and \( \theta_2 \) should be counted counterclockwise from the \( x \) axis.
For the steel ball in question, \( m = 3.00 \text{ kg}, v_1 = v_2 = 10.0 \text{ m/s}, \) and the angles are \( \theta_2 = 90^\circ + 60^\circ = 150^\circ \) and \( \theta_1 = 90^\circ - 60^\circ = 30^\circ \). Therefore,

\[
I_x = 3.00 \text{ kg} \times 10.0 \text{ m/s} \times (\cos 150^\circ - \cos 30^\circ) = -52.0 \text{ kg} \cdot \text{m/s} \quad (\text{S.11})
\]

while

\[
I_y = 3.00 \text{ kg} \times 10.0 \text{ m/s} \times (\sin 150^\circ - \sin 30^\circ) = 0 \quad (\text{S.12})
\]

because \( \sin 150^\circ = \sin 30^\circ = \frac{1}{2} \). Thus, the impulse is in the negative \( x \) direction, perpendicular to and away from the wall.

For a constant force \( \vec{F} \), the impulse is \( \Delta t \times \vec{F} \). For a variable force, we have a similar relation between the impulse and the time-averaged force \( \vec{F}_{\text{avg}} \), namely

\[
\vec{I} = \Delta t \times \vec{F}_{\text{avg}} \quad (\text{S.13})
\]

Thus given the impulse and the time interval, we may find the average force as

\[
\vec{F}_{\text{avg}} = \frac{\vec{I}}{m}. \quad (\text{S.14})
\]

For the steel ball in question, this average force points in the negative \( x \) direction — perpendicular to and away from the wall — and magnitude

\[
F_{\text{avg}} = \frac{I}{m} \frac{52.0 \text{ kg} \cdot \text{m/s}}{0.200 \text{ s}} = 260 \text{ N}. \quad (\text{S.15})
\]

**Textbook problem 6.18:**

Consider the man together with everything he has brought with him. If the ice is so slippery there is no friction at all, then there are no *external* horizontal forces on this system and hence the net horizontal momentum is conserved. Specifically, since initially there is no motion and thus no momentum, the net momentum of the man and everything he brought with him stays zero.
To get out of the frozen pond despite zero net momentum, the man has to throw something away and use recoil. When he throws the book, he gives it momentum $\vec{p}_b = m_b \vec{v}_b$, and since the net momentum stays zero, he himself acquires an equal and opposite momentum

$$\vec{p}_m = -\vec{p}_b \iff M_m \vec{v}_m = -m_b \vec{v}_b. \tag{S.16}$$

Consequently, his velocity vector is

$$\vec{v}_m = -\frac{m_b}{m_m} \times \vec{v}_b. \tag{S.17}$$

The book has mass $m_b = 1.2$ kg while the man has mass $m_m = W_m/g = 730 \text{ N}/9.8 \text{ m/s}^2 \approx 75$ kg; presumably this includes his clothes and everything else he has except the book. This, given the book’s speed $v_b = 5.0 \text{ m/s}$, the man’s recoil speed is

$$v_m = \frac{m_b}{m_m} \times v_b = \frac{1.2 \text{ kg}}{75 \text{ kg}} \times 5.0 \text{ m/s} = 0.080 \text{ m/s}. \tag{S.18}$$

Moving at this speed, the man has to travel $L = 5.0 \text{ m}$ from the pond’s center to its edge. The time this takes is obviously

$$t = \frac{L}{v_m} = \frac{5.0 \text{ m}}{0.080 \text{ m/s}} = 62 \text{ s}, \tag{S.19}$$

or just a bit over one minute.

**Textbook problem 6.20:**

(a) During the brief time when the bullet accelerates down the barrel of the gun, we may neglect all external forces (such as gravity); only the force between the bullet and the gun is big enough to produce a non-negligible impulse during this time. Therefore, the net momentum of the bullet and the gun is conserved during this time period. In other words, the net momentum immediately after the bullet leaves the gun stays the same as it was immediately before the
gun was fired, namely zero. Or in terms of the bullet’s velocity \( \vec{v}_b \) and the gun’s recoil velocity \( \vec{v}_g \),

\[
m_b \vec{v}_b + m_g \vec{v}_g = \vec{p}_{\text{net}}^{\text{after}} = \vec{p}_{\text{net}}^{\text{before}} = \vec{0},
\]

(S.20)

and consequently

\[
\vec{v}_g = -\frac{m_b}{m_g} \times \vec{v}_b.
\]

(S.21)

Numerically, the magnitude of the gun’s recoil velocity is

\[
v_g = \frac{5.0 \text{ g}}{30 \text{ N}/9.8 \text{ m/s}^2} \times 300 \text{ m/s} = 0.49 \text{ m/s}.
\]

(S.22)

(b) If the shooter hold the gun firmly enough to prevent it from moving back, then during the shooting there is a big force between the shooter and the gun, and the impulse of this force cannot be neglected. Consequently, instead of a two-body system of the bullet and the gun we should consider a three-body system of including the bullet, the gun, and the shooter’s own body. The forces external to this system are not impulsive, so the net momentum of the three bodies is conserved during the shooting. Before the shooting, none of these three bodies move, their momenta are all zero, therefore

\[
m_b \vec{v}_b + m_g \vec{v}_g + m_s \vec{v}_s = \vec{p}_{\text{net}}^{\text{after}} = \vec{p}_{\text{net}}^{\text{before}} = \vec{0}.
\]

(S.23)

Moreover, firm hold of the shooter on the gun means that they recoil at the same velocity \( \vec{v}_s = \vec{v}_g \). Hence,

\[
m_b \vec{v}_b + (m_g + m_s) \vec{v}_g = \vec{0} \implies \vec{v}_s = \vec{v}_g = -\frac{m_b}{m_g + m_s} \times \vec{v}_b.
\]

(S.24)

Numerically \( m_g + m_s = (W_g + W_s)/g = (30 \text{ N} + 700 \text{ N})/9.8 \text{ m/s}^2 \approx 75 \text{ kg} \), thus the recoil speed is

\[
v_s = v_g = \frac{5.0 \text{ g}}{75 \text{ kg}} \times 300 \text{ m/s} = 0.02 \text{ m/s}.
\]

(S.25)
Textbook problem 6.46:
This is a recoil problem in relatively slow motion: The heart ejects mass \(m_b\) of blood in one direction while the heart itself — and the rest of the body with it — recoils in the opposite direction. Normally, we do not see this recoil because impulses of various external forces acting on a human body change its momentum by much larger amounts than the blood’s momentum. But a pallet floating on a film of air isolates the patient from all the external horizontal forces, so the net horizontal momentum of the patient (and the pallet) remains exactly constant.

Specifically, the net horizontal momentum stays exactly zero. (Otherwise, the center of mass would be moving at constant velocity in some direction.) When mass \(m_b\) of blood flows down the aorta with velocity \(\vec{v}_b\), it has momentum \(\vec{p}_b = m_b\vec{v}_b\), and this momentum must be canceled by the equal and opposite momentum \(\vec{p}_p = -\vec{p}_b\) of the rest of the patient’s body (and the pallet). Assuming the patient lies still and no part of his body except the blood moves relative to the pallet, we have \(\vec{p}_p = M_p\vec{v}_p\) where \(M_p\) is the combined mass of the patient and the pallet and \(\vec{v}_p\) is their common velocity. (In principle, we should subtract \(m_b\) from the patient’s mass, but since \(m_b \ll M_p\) we may ignore this small correction.) Thus,

\[
M_p\vec{v}_p + m_b\vec{v}_b = \vec{p}_{\text{net}} = \vec{0}
\]

and therefore

\[
m_b v_b = M_p v_p.
\]

In this problem, we know the patient’s mass \(M_p = 54.0\) kg (including the pallet) and the blood’s speed \(v_b = 0.50\) m/s, but we do not know the mass \(m_b\) of the moving blood. Instead, we are given the measurement of the patient+pallet recoil speed

\[
v_p = \frac{6.00 \cdot 10^{-5} \text{ m}}{0.160 \text{ s}} = 3.75 \cdot 10^{-4} \text{ m/s}.
\]

Consequently, we may solve eq. (S.27) for the moving blood’s mass as

\[
m_b = M_p \times \frac{v_p}{v_b} = 54.0 \text{ kg} \times \frac{3.75 \cdot 10^{-4} \text{ m/s}}{0.50 \text{ m/s}} = 40 \text{ g}.
\]
Textbook problem 6.30:
Let’s divide the history of the block and the bullet into two distinct stages. First, the bullet collides with the block and becomes stuck in it. Second, the block (with the bullet inside) slides off the table and falls to the floor. The two stages are governed by different physical laws, so let’s focus on one stage at a time.

The first stage is an inelastic collision. It happens fast, and there are no impulsive forces except between the bullet and the block. Consequently, the net momentum is conserved during the collision, thus

\[ m_b v'_b + M_B v'_B = m_b v_b + M_B v_B. \]  
(S.30)

(In my notations, \( m_b \) and \( v_b \) belong to the bullet while \( M_B \) and \( v_B \) belong to the block.) Moreover, before the collision the block does not move, \( v_B = 0 \), while after the collision, the block and the bullet move together, thus \( v'_b = v'_B = v' \). Consequently, eq. (S.30) becomes

\[ (M_B + m_b) \times v' = m_b \times v_b. \]  
(S.31)

During the second stage, the block (with the bullet in it) slides off the table and fall to the floor. It goes off the table with a horizontal initial velocity \( v_0 = v' \) — which is the velocity it got after the collision — and then we have a projectile motion with initial data

\[ v_{0x} = v', \quad v_{0y} = 0, \quad x_0 = 0, \quad y_0 = H = 1.00 \text{ m}. \]  
(S.32)

Consequently

\[ x(t) = v't \quad \text{and} \quad y(t) = H - \frac{1}{2}gt^2, \]  
(S.33)

the block hits the floor when

\[ y(t_f) = H - \frac{1}{2}gt^2 = 0 \quad \Rightarrow \quad t_f = \sqrt{\frac{2H}{g}}, \]  
(S.34)

and this happens at

\[ x_f = v't_f = v' \times \sqrt{\frac{2H}{g}}. \]  
(S.35)

In this problem, we do not know the bullet’s velocity \( v_b \) before the collision — that’s what we want to find out. Instead, we know that the block lands at \( x_f = 2.00 \text{ m} \), and that allows
us to find the velocity $v'$ of the block when it went off the table: Solving eq. (S.35) for the $v'$, we have
\[
v' = \frac{xf}{\sqrt{2H/g}} = \frac{2.00 \text{ m}}{\sqrt{2(1.00 \text{ m})/(9.8 \text{ m/s}^2)}} = 4.43 \text{ m/s}.
\]

Given this speed, we may now go back to the inelastic collision and solve eq. (S.31) for the bullet’s speed before it hit the block:
\[
v_b = v' \times \frac{M_B + m_b}{m_b} = 4.43 \text{ m/s} \times \frac{250 \text{ g} + 8 \text{ g}}{8.00 \text{ g}} = 143 \text{ m/s}.
\]  

(S.36)

Textbook problem 6.34:

On a frictionless track, there are no external horizontal forces on the three carts, only the internal forces between them. Consequently, the net horizontal momentum of the three carts is conserved, thus
\[
m_1v'_1 + m_2v'_2 + m_3v'_3 = p_{\text{net}} = m_1v_1 + m_2v_2 + m_3v_3.
\]

(S.37)

It does not matter if all three carts collide and stick together all at once, or if two carts collide first with each other and only later with the third cart. One way or the other, their velocities become equal, $v'_1 = v'_2 = v'_3 = v'$, and hence
\[
p_{\text{net}} = (m_1 + m_2 + m_3) \times v'.
\]

(S.38)

Consequently,
\[
v' = \frac{p_{\text{net}}}{m_1 + m_2 + m_3} = \frac{m_1v_1 + m_2v_2 + m_3v_3}{m_1 + m_2 + m_3}
\]
\[
= \frac{4.0 \text{ kg} \times (+5.0 \text{ m/s}) + 10.0 \text{ kg} \times (+3.0 \text{ m/s}) + 3.0 \text{ kg} \times (-4.0 \text{ m/s})}{4.0 \text{ kg} + 10.0 \text{ kg} + 3.0 \text{ kg}} = 2.2 \text{ m/s}.
\]
Textbook problem 6.40:
In a perfectly elastic head-on collision of two bodies, their velocities after the collision are given by

\[
\begin{align*}
v_1' &= \frac{m_1 - m_2}{m_1 + m_2} \times v_1 + \frac{2m_1}{m_1 + m_2} \times v_2, \\
v_2' &= \frac{2m_2}{m_1 + m_2} \times v_1 + \frac{m_2 - m_1}{m_1 + m_2} \times v_2,
\end{align*}
\]

(S.40)

cf. equations (17) and (18) of the supplementary notes. For two bodies of equal masses \(m_1 = m_2\) — such as the two billiard balls in question — eqs. (S.40) simplify to

\[
v_1' = v_2, \quad v_2' = v_1.
\]

(S.41)

In other words, the two bodies exchange velocities.

Thus, in part (a) the first ball stops moving while the second ball starts moving at \(v_2' = +1.5\) m/s away from the first ball. In part (b) both balls reverse their directions of motion: the first ball bounces off at \(v_1' = -1\) m/s while the second bounces off at \(v_2' = +1.5\) m/s. In part (c), both balls continue moving in the same direction, but the first ball slows down to \(v_1' = +1\) m/s while the second ball speeds up to \(v_2' = +1.5\) m/s. (Sign convention: the initial velocity of the first ball is positive, \(v_1 = +1.5\) m/s.)

Textbook problem 6.42:
(a) We do not know if this collision is elastic, inelastic, or partially elastic, but we do know that the net momentum of the two objects is conserved. Thus

\[
m_1 \vec{v}_1' + m_2 \vec{v}_2' = m_1 \vec{v}_1 + m_2 \vec{v}_2,
\]

(S.42)

and therefore

\[
\vec{v}_2' = \vec{v}_2 + \frac{m_1}{m_2} \times (\vec{v}_1 - \vec{v}_1'),
\]

(S.43)

or in components

\[
\begin{align*}
v_{2x}' &= v_{2x} + \frac{m_1}{m_2} \times (v_{1x} - v_{1x}) , \\
v_{2y}' &= v_{2y} + \frac{m_1}{m_2} \times (v_{1y} - v_{1y}).
\end{align*}
\]

(S.44)

Let the \(x\) axis point due East while the \(y\) axis points due North. In these coordinates,
before the collision

\[ v_{1x} = 15.0 \text{ m/s}, \quad v_{1y} = 0, \quad \text{and} \quad v_{2x} = v_{2y} = 0. \]  
(S.45)

After the collision, we know that

\[ v'_{1x} = 0 \quad \text{and} \quad v'_{1y} = -4 \text{ m/s}. \]  
(S.46)

Hence, according to eqs. (S.44),

\[
\begin{align*}
v'_{2x} &= 0 + \frac{8.0 \text{ kg}}{10.0 \text{ kg}} \times (15.0 \text{ m/s} - 0) = +12.0 \text{ m/s}, \\
v'_{2y} &= 0 + \frac{8.0 \text{ kg}}{10.0 \text{ kg}} \times (0 - (-4.00 \text{ m/s})) = +3.20 \text{ m/s}.
\end{align*}
\]  
(S.47)

Thus, after the collision, the second object moves at speed

\[ v'_{2} = \sqrt{(v'_{2x})^2 + (v'_{2y})^2} = \sqrt{(12.0 \text{ m/s})^2 + (3.20 \text{ m/s})^2} = 12.4 \text{ m/s} \]  
(S.48)

in the direction

\[ \theta = \arctan \frac{v'_{2y}}{v'_{2x}} = \arctan \frac{3.20 \text{ m/s}}{+12.0 \text{ m/s}} \approx 15^\circ, \]  
(S.49)

i.e., 15° North from due East.

(b) Before the collision, the net kinetic energy of the two objects was

\[ K = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}(8.00 \text{ kg})(15.0 \text{ m/s})^2 + 0 = 900 \text{ J}. \]  
(S.50)

After the collision, the net kinetic energy is

\[ K' = \frac{1}{2}m_1v'_{1}^2 + \frac{1}{2}m_2v'_{2}^2 = \frac{1}{2}(8.00 \text{ kg})(4.00 \text{ m/s})^2 + \frac{1}{2}(10.0 \text{ kg})(12.4 \text{ m/s})^2 = 825 \text{ J}. \]  
(S.51)

The remaining 900 J = 825 J = 75 J were lost in the collision, which means the collision was not perfectly elastic. (But it was not totally inelastic either, because the two objects bounce off in different directions.)

Relative to the initial 900 J of kinetic energy, the energy loss of 75 J is \( \frac{1}{12} \), or about 8.33%. 

10
Textbook problem 6.44:
This is a totally inelastic collision, after which the two cars stick together and move with a common velocity $\vec{v}'$. The net momentum is conserved in any collision, therefore

$$(m_1 + m_2)\vec{v}' = \vec{p}_{\text{net}} = m_1\vec{v}_1 + m_2\vec{v}_2.$$  (S.52)

And since the two cars in questions have equal masses,

$$2\vec{v}' = \vec{v}_1 + \vec{v}_2.$$  (S.53)

In this problem we know the directions of all the velocity vectors before and after the collision, so the vector diagram for eq. (S.53) looks like

Solving this right triangle we immediately have

$$\left|\frac{\vec{v}_2}{\vec{v}_1}\right| = \tan 55^\circ$$  (S.54)

and hence

$$v_2 = v_1 \times \tan 55^\circ = 13.0 \text{ m/s} \times 1.43 = 18.6 \text{ m/s}.$$  (S.55)

Converting the two car’s speeds into miles per hour, we find

$$v_1 = 29 \text{ mi/hr}, \quad v_2 = 41.5 \text{ mi/hr}.$$  (S.56)

the first car was going well under the speed limit, but the second car was definitely speeding.
Non-textbook problem #1:
All starts, planets, and large enough moons and asteroids are spheres or slightly oblate spheroids, so by symmetry their center of mass are located in their geometric center. In particularly, Pluto’s own CM is at Pluto’s center, and Charon’s CM is at Charon’s center, $L = 19600$ km away from Pluto’s. Consequently, to find the CM of the Pluto–Charon system, we may treat both bodies as point-like particles with all mass concentrates at the respective CM’s. Consequently, the system’s CM lies on the straight line connecting Pluto’s and Charon’s centers, and its distance from Pluto’s center is

$$L_P = \frac{M_C}{M_P + M_C} \times L = \frac{1.9 \cdot 10^{21} \text{ kg}}{1.27 \cdot 10^{22} \text{ kg} + 1.9 \cdot 10^{21} \text{ kg}} \times 19,600 \text{ km} = 2550 \text{ km}. \quad (S.57)$$

This distance is larger than Pluto’s radius $R_P = 1137$ km, so the system’s CM is above Pluto’s surface by $H = L_P - R_P \approx 1410$ km.

Non-textbook problem #2:
The only external horizontal force on the man+boat system is the water drag on the boat’s hull. When the boat moves slowly (relative to the water), the water drag force is small and may be neglected, which leaves with no horizontal external forces at all. Consequently, the net momentum of the man+boat system is conserved,

$$\vec{p}^\text{net} = (M_b + M_m) \vec{v}_\text{cm} = \text{const.} \quad (S.58)$$

And since initially neither man nor boat were moving, the net momentum was zero, and it stays zero while the man walks on the boat. The relation between the net momentum and the center of mass’s velocity, zero net momentum means $\vec{v}_\text{cm} = \vec{0}$, or in other words, the center of mass stays motionless. Therefore, while the man walks towards the pier, the boat has to move away to keep their combined CM at the same place.

To find the magnitude of the boat’s displacement, we need to relate it to displacements of the man and of the man+boat system’s CM. In terms of the man’s CM located at $X^\text{man}_\text{cm}$
and the boat’s CM located at $X_{\text{cm}}^{\text{boat}}$, the system’s CM is at

$$X_{\text{cm}}^{\text{system}} = \frac{M_m}{M_m + M_b} \times X_{\text{cm}}^{\text{man}} + \frac{M_b}{M_m + M_b} \times X_{\text{cm}}^{\text{boat}},$$  \hspace{1cm} (S.59)

so its displacement is

$$\Delta X_{\text{cm}}^{\text{system}} = \frac{M_m}{M_m + M_b} \times \Delta X_{\text{cm}}^{\text{man}} + \frac{M_b}{M_m + M_b} \times \Delta X_{\text{cm}}^{\text{boat}}.$$  \hspace{1cm} (S.60)

We do not know where the boat’s CM is relative to the boat’s stern or prow. But since the boat does not change shape or rotate, the displacement of every particular part of the boat is exactly the same,

$$\Delta X^{\text{prow}} = \Delta X^{\text{stern}} = \cdots = \Delta X^{\text{boat}},$$  \hspace{1cm} (S.61)

and therefore the CM’s displacement is simply the boat’s displacement,

$$\Delta X_{\text{cm}}^{\text{boat}} = \Delta X_{\text{cm}}^{\text{boat}}.$$  \hspace{1cm} (S.62)

As to the man, his legs and torso do not exactly move together as he stand up and walks from the stern to the prow, but those differences are small compared to his overall displacement from the stern to the prow. Consequently, we may neglects the man’s horizontal size (and the fact that he moves one foot at a time) and approximate

$$X_{\text{cm}}^{\text{man}} = X^{\text{man}} \quad \text{and} \quad \Delta X_{\text{cm}}^{\text{man}} = \Delta X^{\text{man}}.$$  \hspace{1cm} (S.63)

Therefore,

$$\Delta X_{\text{cm}}^{\text{system}} = \frac{M_m}{M_m + M_b} \times \Delta X_{\text{cm}}^{\text{man}} + \frac{M_b}{M_m + M_b} \times \Delta X^{\text{boat}}.$$  \hspace{1cm} (S.64)

On the other hand, we know that the net momentum of the man+boat system stays zero and therefore the system’s CM does not move at all. Thus,

$$\Delta X_{\text{cm}}^{\text{system}} = 0,$$  \hspace{1cm} (S.65)

and combining this with eq. (S.64), we arrive at

$$M_m \times \Delta X_{\text{cm}}^{\text{man}} + M_b \times \Delta X^{\text{boat}} = (M_m + M_b) \times \Delta X_{\text{cm}}^{\text{system}} = 0.$$  \hspace{1cm} (S.66)

Eq. (S.66) gives us one relation between displacements of the man and the boat relative
to the pier. We also know that the man starts at the boat’s stern and ends at its prow, which gives us his displacement relative to the boat, $\Delta X^{\text{rel}} = L = 12 \text{ ft}$, the length of the boat. In terms of the man’s and the boat’s displacements relative to the pier,

$$\Delta X^{\text{rel}} = \Delta X^{\text{man}} - \Delta X^{\text{boat}},$$  \hfill (S.67)

which tells us that

$$\Delta X^{\text{man}} - \Delta X^{\text{boat}} = L = 12 \text{ ft}.$$  \hfill (S.68)

Together, eqs. (S.66) and (S.68) give us two linear equations for two unknowns, and all we need to do is to solve them for the boat’s displacement. Combining the two equations, we have

$$\Delta X^{\text{boat}} + L = \Delta X^{\text{man}} = -\frac{M_b}{M_m} \times \Delta X^{\text{boat}},$$  \hfill (S.69)

hence

$$\left(1 + \frac{M_b}{M_m}\right) \times \Delta X^{\text{boat}} = -L,$$  \hfill (S.70)

and therefore

$$\times \Delta X^{\text{boat}} = -L \times \left(1 + \frac{M_b}{M_m}\right)^{-1}$$

$$= -12 \text{ ft} \times \left(1 + \frac{70 \text{ lb}}{140 \text{ lb}}\right)^{-1}$$

$$= -12 \text{ ft} \times \left(\frac{3}{2}\right)^{-1}$$

$$= -8 \text{ ft}.$$  \hfill (S.71)

Thus, while the man walks 12 feet from the stern to the prow, the boat moves back 8 feet (i.e., 8 feet away from the pier).

And by the way, the man’s displacement relatively to the pier is only $\Delta X^{\text{man}} = 12 \text{ ft} - 8 \text{ ft} = +4 \text{ ft}.$