Problem **I**:

When a unit of something is prepended with suffix "micro" it means one millionth of that unit.* A century is a unit of time equal to 100 years, hence a micro-century is one millions of a century,

$$1\,\mu\text{century} \;=\; \frac{1}{1000000}\times 100\,\text{years} \;=\; 0.0001\,\text{year}.$$

To compare this period in minutes, we note that $1\,\text{year} = 365.25\,\text{days}$, $1\,\text{day} = 24\,\text{hours}$, and $1\,\text{hour} = 60\,\text{minutes}$, thus

$$\begin{aligned}
1\,\mu\text{century} &= 0.0001\,\text{year}\times 365.25\,\frac{\text{days}}{\text{year}} \;=\; 0.036525\,\text{days}\\
&= 0.036525\,\text{days}\times 24\,\frac{\text{hours}}{\text{day}} \;=\; 0.8766\,\text{hours}\\
&= 0.8766\,\text{hours}\times 60\,\frac{\text{minutes}}{\text{hour}} \;=\; 52.596\,\text{minutes}\\
&\approx 52.6\,\text{minutes}.
\end{aligned}$$

In other words, Fermi's lecture was just a couple of minutes longer then a regular 50-minute class.

Problem **II**:

Let's convert everything to metric units. A furlong is 220 yards, a yard is 3 feet, a foot is 0.3048 meter, thus

$$1\text{ furlong} \;=\; 220\times 3\times 0.3048\text{ m} \;=\; 201.168\text{ m}.$$

A fortnight is 14 day&nights of 24 hours each, and an hour is 3600 seconds, thus

$$1\text{ fortnight} \;=\; 14\times 24\times 3600\text{ s} \;=\; 1209600\text{ s}.$$

⋆ In other context, "micro-" simply means small. For example, a micro-brewery is a small brewery rather than one millions of a brewery. But that's because a brewery isn't a unit of some quantity.

Hence, a furlong per fortnight is a unit of speed equal to

$$1\,\frac{\text{furlong}}{\text{fortnight}} \;=\; \frac{1\text{ furlong}}{1\text{ fortnight}} \;=\; \frac{201.168\,\text{m}}{1209600\,\text{s}} \;=\; \frac{201.168}{1209600}\,\text{m/s} \;\approx\; 1.663\times10^{-4}\,\text{m/s}.$$

So converting the naturalist's report of snail's average speed to metric units we get 1.663×10^{-4} m/s $= 0.1663$mm/s. But of course we should not write down so many significant digits because the original data (1 furlong/fortnight) wasn't so precise; instead, we should round-off the metric value down to one or two significant digits, thus $v \approx 0.17$mm/s or $\frac{1}{6}$ millimeter per second.

At this average speed, the snail would crawl in $t = 1\,\text{minute} \;=\; 60\,\text{s}$ through the distance

$$L \;=\; v\times t \;=\; 0.17\,\text{mm/s}\times 60\,\text{s} \;=\; 10.2\,\text{mm} \;\approx\; 10\,\text{mm} \;=\; 1\,\text{cm}.$$

In other words, in one minute the snail would crawl through approximately one centimeter.

Problem **III**:

Geometrically, each yellow line is a slab of paint with extremely different dimensions:

$$\begin{aligned} \text{length}\quad a &= 100\text{ km} = 100\times10^3\text{ m} = 10^5\text{ m},\\ \text{width}\quad b &= 10\text{ cm} = 10\times10^{-2}\text{ m} = 10^{-1}\text{ m},\\ \text{thickness}\quad c &= 1\text{ mm} = 1\times10^{-3}\text{ m} = 10^{-3}\text{m}. \end{aligned} \tag{1}$$

The volume of this slab is a product

$$V \;=\; a\times b\times c, \tag{2}$$

but using this formula requires consistent units: the unit of volume V must be a product of units of linear dimensions a, b, and c. For example, if a, b, and c are all in meters, then the volume comes up in cubic meters,

$$V \;=\; (10^5\text{ m})\times(10^{-1}\text{ m})\times(10^{-3}\text{ m}) \;=\; 10^{5-1-3}\text{ m}^3 \;=\; 10\text{ m}^3. \tag{3}$$

On the other hand, if a is in kilometers, b is in centimeters, and c is in millimeters, then the volume

$$V \;=\; (100\text{ km})\times(10\text{ cm})\times(1\text{ mm}) \;=\; 1000\text{ km}\cdot\text{cm}\cdot\text{mm} \tag{4}$$

comes out in strange units of km · cm · mm.

To keep our units simple, in eq. (1) we have converted a, b, and c into similar units, namely meters, and then eq. (3) gave us the volume in cubic meters. After that, we can convert this volume into any units we like. Since the problem asks about liters, we convert from cubic meters into liters as

$$V = 10\ \text{m}^3 \times (1000\ \text{L/m}^3) = 10,000\ \text{L}. \tag{5}$$

Note that this is the volume of a single yellow line, or rather, the volume of paint making that line when the paint was wet. In other words, painting that line took 10,000 liters of paint. Painting the second line took another 10,000 liters, for the total of 20,000 liters of paint (about 5300 gallons).

Problem **IV**:

(a) First, since all the balls are made from the same material, they have equal *densities,*

$$\text{density} = \frac{\text{mass}}{\text{volume}} = \text{const.} \tag{6}$$

In other words, the mass M of such a ball is *proportional* to its volume V,

$$M \propto V. \tag{7}$$

Second, for any fixed geometric shape, the volume of a body is proportional to its linear size,

$$V \propto (\text{size})^3. \tag{8}$$

For example, a cube with side a has volume $V_c = a^3$, a tetrahedron with side a has $V_t = \frac{\sqrt{2}}{12}\,a^3$, a pyramid with height = side = a has $V_p = \frac{1}{3}\,a^3$, *etc.*, *etc.* In particular, a ball of radius R has volume

$$V_b = \frac{4\pi}{3}\,R^3. \tag{9}$$

For this problem, we don't need the coefficient $4\pi/3$, all we need to know is that the volume of a ball is proportional to the cube of its radius,

$$V \propto R^3. \tag{10}$$

Finally, combining eqs. (7) and (10), we immediately see that the mass of a ball is proportional to the cube of its radius,

$$M \propto V \propto R^3. \tag{11}$$

(b) The proportionality (11) means that the ratio M/R^3 is the same for all the balls in question: For any two balls,

$$\frac{M_1}{R_1^3} = \frac{M_2}{R_2^3}. \tag{12}$$

Algebraically, this relation is equivalent to

$$\frac{M_2}{M_1} = \frac{R_2^3}{R_1^3} = \left(\frac{R_2}{R_1}\right)^3. \tag{13}$$

Thus, if the second ball happens to have twice the radius of the first ball, $R_2 = 2R_1$, their mass ratio is

$$\frac{M_2}{M_1} = \left(\frac{R_2}{R_1} = 2\right)^3 = 8. \tag{14}$$

In other words, the second ball is $2^3 = 8$ times heavier than the first ball. Given the first ball's mass $M_1 = 1$ kg, the second ball has mass $M_2 = 8 \times 1$ kg $= 8$ kg.

Problem **V**:

On his $20^{\underline{\text{th}}}$ birthday, you friend's age is 20 years or $20 \times 365\frac{1}{4} = 7305$ days, *give or take a fraction of a day.* (This uncertainty reflects that you know the year and the day of his birth, but not the hour.) This age is accurate to four significant figures, so when you convert it from days to other units of time, you should keep four figures and round off the rest.

Converting to seconds, we have 1 day $=$ 24 hr $\times$ 60 min/hr $\times$ 60 s/min $= 24 \times 60 \times 60$ s $= 86,400$ seconds. Hence your friend's age in seconds is $T = 7305$ days $\times$ $86,400$ s/day $= 631,152,000$ second. Rounding this number to four significant figures, we write it as 631,200,000 seconds, or in scientific notations as $6.312 \cdot 10^8$ seconds.

Problem **VI**:

Obviously, the net weight of the student and his backpack is $W = 150\text{ lb} + 17.3\text{ lb} = 167.3\text{ lb}$. The real question is, what's the accuracy of this number and how may unreliable figures we should round off.

The accuracy of a sum (or a difference) is controlled by the less accurate of the terms. But what's relevant here is not the numbers of significant figures in each terms but where they are with respect to the decimal point. The backpack's weight 17.3 pounds is accurate to the first figure to the right of the decimal point. On the other hand, the student's own weight of 150 pounds is accurate only to the second figure (5) to the left of the point. The zero in 150 is not a significant figure — it's there only to indicate where the decimal point is — so the student weight could be anywhere between 145 and 155 pounds.

Since the backpack's weight is known to a higher accuracy, the uncertainty of the net weight follows from the uncertainty of the student weight — give or take up to 5 pounds. Consequently, only the two leftmost figures (1 and 6) of the net weight $W = 167.3$ lb are significant, the rest should be rounded off. Thus, using the right number of significant figures, *the net weigh of the student and his backpack is 170 pounds.*

Problem **VII** is postponed to the homework set #2.