Textbook question **Q4** (chapter 2):

(a) By definition

$$\text{average speed} = \frac{d}{t} = \frac{\text{distance covered}}{\text{time (including stops)}}.$$

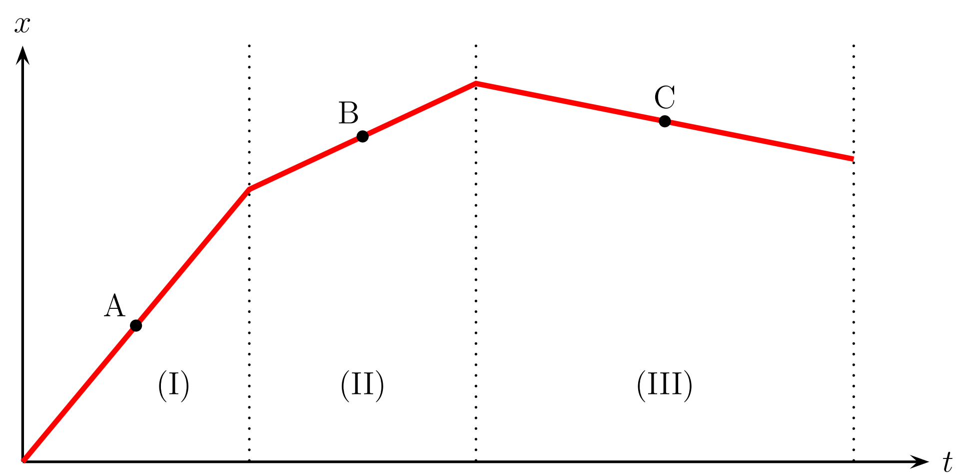
The hare and the tortoise run over the same net distance d, but the hare takes longer time because of his stops, $t_{\text{hare}} > t_{\text{tortoise}}$. Consequently, his average speed is slower than the tortoise's:

$$v_{\text{hare}}^{\text{av}} = \frac{d}{t_{\text{hare}}} < \frac{d}{t_{\text{tortoise}}} = v_{\text{tortoise}}^{\text{av}}.$$

(b) The tortoise plods steadily forward, so it's instantaneous speed is approximately constant during the race, $v_{\text{tortoise}}(t) \approx v_{\text{tortoise}}^{\text{av}}$ at all times t. But the hare's instantaneous speed $v_{\text{hare}}(t)$ varies a lot with time: at some t he stops altogether (thus $v_{\text{hare}}(t) = 0$) while at other t he runs very fast, much faster than the tortoise, (thus $v_{\text{hare}}(t) \gg v_{\text{tortoise}}(t)$). So his *maximal instantaneous speed* during the race is much faster then the tortoise's.

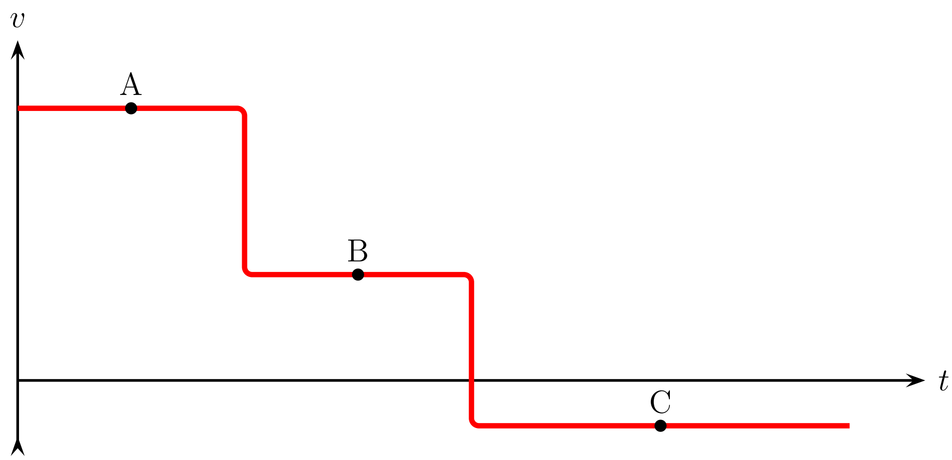
Textbook questions **Q22** and **Q23** (chapter 2):

On the motion graph position x versus time t, the instantaneous velocity at some point is the slope of the tangent line to the $x(t)$ curve at that point. Let's apply this rule to the car's motion plotted on diagram 22 (page 35); here is this diagram (with some extra marking):



The curve $x(t)$ looks like it's made of three approximately straight segments for time intervals

(I), (II), and (III). Consequently, *during each time interval, the car's instantaneous velocity is approximately constant* — which answers question **Q23** — *but it's a different constant for each interval!* Graphically, car's instantaneous velocity $v(t)$ as a function of time looks likes the following plot:



In particular, the instantaneous velocity during the third interval (III) is negative — according to the negative (downward) slope of the $x(t)$ curve on the previous diagram — which means that the car is moving backward. This answers question **Q22**(a). During the other two intervals (I) and (II), the velocity is positive, but during the first interval the velocity is larger than during the second interval. In particular, car's (instantaneous) velocity at point A is larger than at point B, $v(A) > v(B)$ — which answers question **Q22**(b).

Non-textbook problem **I**:

Let's divide the wrong-way trip from Austin to San Antonio into 3 segments: (1) 100 miles from Austin to Waco; (2) 100 miles from Waco back to Austin; (3) 80 miles from Austin to San Antonio. To get the total distance traveled during this trip, we add up the 3 segment's lengths *arithmetically,*

$$L \;=\; 100\text{ miles} \;+\; 100\text{ miles} \;+\; 80\text{ miles} \;=\; 280\text{ miles}. \tag{1}$$

And the average speed for the trip is this total distance divided by the trip's time $t = 4$ hours,

$$\text{average speed} \;=\; \frac{L}{t} \;=\; \frac{280\text{ miles}}{4\text{ hours}} \;=\; 70\text{ MPH}. \tag{2}$$

To get the net displacement of the trip, we add its 3 segments *algebraically,* with the sign of each segment depending on its direction. Taking the northbound direction to be positive, we have

$$\Delta X \;=\; (+100\ \text{miles}) \;+\; (-100\ \text{miles}) \;+ (-80\ \text{miles}) \;=\; -80\ \text{miles}, \tag{3}$$

where the negative sign of this displacement indicates its southward direction. Note that the net displacement does not depend how the driver got from Austin to San Antonio; he could have driven all the way to Canada, then driven back and continued to Mexico, *etc.* — as long as he started in Austin and ended in San Antonio, his net displacement would be the same

$$\Delta X \;=\; X_{\text{San Antonio}} \;-\; X_{\text{Austin}} \;=\; -80\ \text{miles}. \tag{4}$$

Indeed, using the mile marks along the I 35 as coordinates, we have $X_{\text{Austin}} = 235$ (miles), $X_{\text{SanAntonio}} = 155$ (miles), and hence the net displacement precisely as in eq. (4)

Given the net displacement (3), the average velocity for the trip was

$$\text{average velocity} \;=\; \frac{\Delta X}{t} \;=\; \frac{-80\ \text{miles}}{4\ \text{hours}} \;=\; -20\ \text{MPH}, \tag{5}$$

where the minus sign indicates the southward direction of this average velocity. Note that the magnitude of this average velocity is quite smaller than the average speed: This always happens when the trip includes motion in different directions.

PS: If you took the south-bound direction to be positive, you should get opposite signs of the net displacement

$$\Delta X \;=\; (-100\ \text{miles}) \;+\; (+100\ \text{miles}) \;+ (+80\ \text{miles}) \;=\; +80\ \text{miles} \tag{6}$$

and of the net velocity

$$\text{average velocity} \;=\; \frac{\Delta X}{t} \;=\; \frac{+80\ \text{miles}}{4\ \text{hours}} \;=\; +20\ \text{MPH}. \tag{7}$$

I will accept both positive and negative answers as correct, provided you say which direction — northbound or southbound — you choose to be positive, and all your signs are consistent with this choice.

Formulæ for the textbook problems **E14** and **E16**:

For a motion with constant acceleration along a straight line, the velocity v increases or decreases with time t according to

$$v(t) \;=\; v_0 \;+\; at \tag{8}$$

where a is the acceleration and $v_0 = v(t=0)$ is the initial velocity, while the position x changes as

$$x(t) \;=\; x_0 \;+\; v_0 t \;+\; \tfrac{1}{2}at^2 \tag{9}$$

where $x_0 = x(t=0)$ is the initial position. The net displacement from the initial point is

$$\Delta x(t) \;\stackrel{\text{def}}{=}\; x(t) \;-\; x_0 \;=\; v_0 t \;+\; \tfrac{1}{2}at^2, \tag{10}$$

but in class I have derived a simpler formula:

$$\Delta x(t) \;=\; t \times \frac{v(t)+v_0}{2}\,. \tag{11}$$

Textbook problem **E14** (chapter 2):

(a) At time $t=0$, the runner in question has initial velocity $v = 2.0$ m/s. For the next three seconds, he accelerates at the uniform rate $a = 1.1$ m/s^2. His velocity at the end of this 3.0-second period is given by eq. (8):

$$v(t) \;=\; v_0 \;+\; a\times t \;=\; (2.0\text{ m/s}) \;+\; (1.1\text{ m/s}^2)\times(3.0\text{ s}) \;=\; 5.3\text{ m/s}. \tag{12}$$

(b) The distance the runner covers in those 2 seconds can be obtained from eq. (10):

$$\begin{aligned}\Delta x \;&=\; v_0 t \;+\; \tfrac{1}{2}at^2 \;=\; (2.0\text{ m/s})\times(6.0\text{ s}) \;+\; \tfrac{1}{2}(1.1\text{ m/s}^2)\times(3.0\text{ s})^2 \\ &=\; 6.0\text{ m} \;+\; 4.95\text{ m} \;=\; 10.95\text{ m} \;\approx 11\text{ m}.\end{aligned} \tag{13}$$

Alternatively, it can be obtained from eq. (11)

$$\Delta x(t) \;=\; t\times\frac{v(t)+v_0}{2} \;=\; (3.0\,\text{s})\times\frac{5.3\text{ m/s} \,+\, 2.0\text{ m/s}}{2} \;=\; (3.0\,\text{s})\times(3.65\,\text{m/s}) \;\approx\; 11\,\text{m}. \tag{14}$$

Textbook problem **E16** (chapter 2):

This time, the runner has a higher initial velocity $v_0 = 4.0$ m/s, but her acceleration is negative $a = -1.5$ m/s^2. Apart from that difference, this problem uses exactly the same formulæ as the previous problem E14:

(a) After $t = 2.0$ s of deceleration, the runner velocity decreases to

$$v \;=\; v_0 + at \;=\; (4.0\,\mathrm{m/s}) + (-1.5\,\mathrm{m/s}^2)\times(2.0\,\mathrm{s}) \;=\; 4.0\,\mathrm{m/s} - 3.0\,\mathrm{m/s} \;=\; 1.0\,\mathrm{m/s}. \qquad (15)$$

(b) The distance she covers during this time is given by eq. (10) or eq. (11):

$$\Delta x \;=\; v_0 t + \tfrac{1}{2}at^2 \;=\; (4.0\,\mathrm{m/s})\times(2.0\,\mathrm{s}) + \tfrac{1}{2}(-1.5\,\mathrm{m/s}^2)\times(2.0\,\mathrm{s})^2 \;=\; 8.0\,\mathrm{m} - 3.0\,\mathrm{m} \;=\; 5.0\,\mathrm{m}, \qquad (16)$$

or equivalently

$$\Delta x(t) \;=\; t\times\frac{v(t)+v_0}{2} \;=\; (2.0\,\mathrm{s})\times\frac{1.0\ \mathrm{m/s}\;+\;4.0\ \mathrm{m/s}}{2} \;=\; (2.0\,\mathrm{s})\times(2.5\,\mathrm{m/s}) \;=\; 5.0\,\mathrm{m}. \qquad (17)$$

Non-textbook problem **II**:

Note: I use a coordinate system where the positive direction is UP and the origin of the x coordinate is on the ground.

In the absence of air resistance, the rock falls with a constant acceleration $a = -g$, where the minus sign indicates the downward direction of this acceleration. Since the rock starts falling with zero initial velocity, its equations of motion are very simple:

$$v(t) \;=\; \cancel{v_0} \;-\; gt, \qquad (18)$$

$$x(t) \;=\; x_0 \;+\; \cancel{v_0}t \;-\; \tfrac{1}{2}gt^2, \qquad (19)$$

where $x_0 = +144$ ft is the initial position of the rock (on the roof).

(a) The rock hits the ground when $x(t) = 0$. According to eq. (19), this happens when

$$x_0 \;-\; \tfrac{1}{2}gt^2 \;=\; 0. \tag{20}$$

To find the time the rock was falling, we simply solve this equation for t:

$$\tfrac{1}{2}gt^2 \;=\; x_0 \quad\Longrightarrow\quad t^2 \;=\; \frac{2x_0}{g} \quad\Longrightarrow\quad t \;=\; \sqrt{\frac{2x_0}{g}}\,. \tag{21}$$

Numerically,

$$t \;=\; \sqrt{\frac{2 \times 144\ \mathrm{ft}}{32\ \mathrm{ft/s^2}}} \;=\; \sqrt{9.0\ \mathrm{s}^2} \;=\; 3.0\ \mathrm{s}. \tag{22}$$

(b) According to eq. (18), after $t = 3.0$ s of falling, the rock reaches velocity

$$v \;=\; -g \times t \;=\; -g \times \sqrt{\frac{2x_0}{g}} \;=\; -\sqrt{2x_0 \times g}\,. \tag{23}$$

The negative sign of this velocity indicates that the rock is moving down rather than up. The speed of the rock is the magnitude of this velocity,

$$|v| \;=\; g \times t \;=\; \sqrt{2x_0 \times g}\,. \tag{24}$$

Numerically, the speed of the rock hitting the ground is

$$|v| \;=\; g \times t \;=\; (32\ \mathrm{ft/s^2}) \times (3.0\ \mathrm{s}) \;=\; 96\ \mathrm{ft/s}. \tag{25}$$

(c) In other units, the speed of a falling rock is

$$\begin{aligned} |v| \;&=\; 96\ \mathrm{m/s} \times 0.3048\ \mathrm{m/ft} \;=\; 29.2608\ \mathrm{m/s}, \\ &=\; (29.2608\ \mathrm{m/s}) \times (3600\ \mathrm{s/hr}) \Big/ (1000\ \mathrm{m/km}) \;=\; 105.34\ \mathrm{km/hr}, \\ &=\; (105.34\ \mathrm{km/hr}) \Big/ (1.609\ \mathrm{km/mile}) \;=\; 65.47\ \mathrm{mile/hr}. \end{aligned} \tag{26}$$

Rounding these numbers off to 2 significant figures, we obtain

$$|v| \;\approx\; 29\ \mathrm{m/s} \;\approx\; 105\ \mathrm{km/hr} \;\approx\; 65\ \mathrm{MPH}. \tag{27}$$