Non-textbook problem #**I**:

See solutions to homework set #4, non-textbook problem VII.

Non-textbook problem #**II**:

As I explained in class, the First Law of Newton is trickier than it's usually written down: *In the absence of forces, a body at rest stays at rest while a body in motion continues to move at constant velocity vector* — but only relative to a reference frame which itself has a constant velocity vector. Such frames are called *inertial*.

The accelerating frames of reference are *non-inertial* — the velocity vector of a body relative to such a frame can change without any forces acting on the body, simply because the frame's own velocity is changing. In particular, a bus that's speeds up, slows down, or turns is a non-inertial frame, and the First Law does not apply to velocities relative to the bus.

On the other hand, the ground is an inertial frame (to a good approximation). So when no forces act on a bag laying on a seat of a moving bus, the bag moves at constant velocity vector *relative to the ground.* But the bag's motion relative to the bus depends on the bus's own motion. Indeed,

$$\vec{\mathbf{v}}^{\text{bag}}_{(\text{bus})} = \vec{\mathbf{v}}^{\text{bag}}_{(\text{ground})} - \vec{\mathbf{v}}^{\text{bus}}_{(\text{ground})}. \tag{1}$$

so if the bag keeps constant velocity relative to the ground but the bus's velocity changes, the bag's velocity relative to the bus would also change.

For example, if the bag and the bus initially move at the same velocity relative to the ground — so the bag's velocity relative to the bus is zero — but then the bus suddenly slows down while the bag continues moving forward at constant velocity (since no forces act on it), the relative velocity would suddenly become positive. Thus, *relative to the bus,* the bag would suddenly slide forward off the seat.

Non-textbook problem #**III**:

On Mars, your *mass* m would be exactly the same as on Earth, but your *weight* $W = mg$ would be different because of a different gravitational field $g(\text{Mars}) \neq g(\text{Earth})$. Specifically,

$$\frac{\text{your weight on Mars}}{\text{your weight on Earth}} = \frac{m \times g(\text{Mars})}{m \times g(\text{Earth})} = \frac{g(\text{Mars})}{g(\text{Earth})} = \frac{3.7 \text{ m/s}^2}{9.8 \text{ m/s}^2} \approx 0.38, \tag{2}$$

in other words, your weight on Mars is 38% of your weight on Earth.

For example, a person whose weight on Earth is $W_E = 140$ lb ≈ 620 N would weigh on Mars only $W_M = 0.38 \times 140 \text{ lb} = 53 \text{ lb} \approx \text{N lb}$.

Non-textbook problem #**IV**:

By the Third Law of Newton, John pulls Mary with the same force F that Mary pulls John. Since there are no other horizontal forces acting on John, Newton's Second Law relates this force to John's mass and acceleration as

$$F = m_J a_J \,. \tag{3}$$

Likewise, there are no other horizontal forces acting on Mary, so her acceleration and mass are also related to the same force,

$$F = m_M a_M \,. \tag{4}$$

Combining the two equations, we obtain

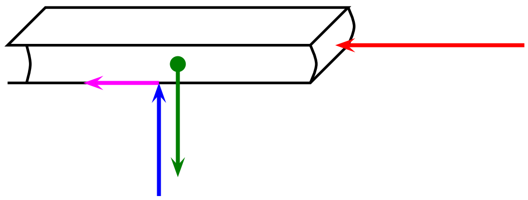
$$m_M a_M = F = m_J a_J, \tag{5}$$

from which we can find Mary's acceleration as

$$a_M = \frac{F}{m_M} = \frac{m_J a_J}{m_M} = \frac{77 \text{ kg} \times 2.5 \text{ m/s}^2}{55 \text{ kg}} = 3.5 \text{ m/s}^2. \tag{6}$$

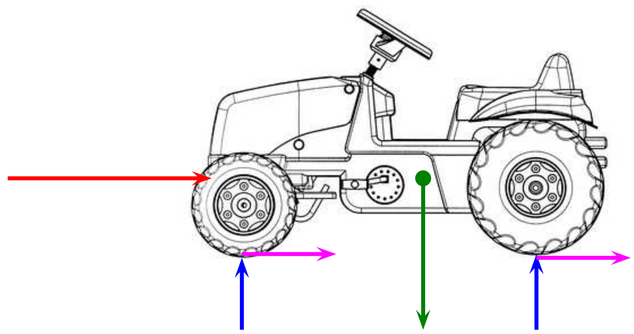
Textbook question **Q29** (chapter 4):

This diagram shows all the forces acting on the book:



The green arrow here is the book's weight $W_b = M_b g$, the blue arrow is the normal force $N^{(\text{table})}_{\text{book}}$ from the table (on which the book lays) to the book, the purple arrow $f^{(\text{table})}_{\text{book}}$ is the friction force from the table on the book, and the red arrow is the force $F^{(\text{tractor})}_{\text{book}}$ with which the tractor pushes the book.

And this diagram shows all the forces acting on the toy tractor:



Here the green arrow is the tractor's weight $W_t = M_t g$, the blue arrows are the normal forces $N^{(\text{table})}_{\text{wheels}}$ from the table on the tractor's wheels, the purple arrows are the friction forces $f^{(\text{table})}_{\text{wheels}}$ from the table on the wheels, and the red arrow is the force $F^{(\text{book})}_{\text{tractor}}$ with which the book pushes back the tractor.

By Newton's Third Law, the book pushes the tractor back with exactly the same force as the tractor pushes the book. Thus, the red arrows on the two diagrams are the action-reaction pair, that's why they have the same lengths but opposite directions. In vector notations,

$$\vec{\mathbf{F}}^{(\text{book})}_{\text{tractor}} = -\vec{\mathbf{F}}^{(\text{tractor})}_{\text{book}}. \tag{7}$$

For the other forces shown on the two diagrams, the corresponding reaction force acts on something other than the book or the tractor. For example, for the green arrows — the gravity forces on the book and on the tractor — the corresponding reaction forces act on the Earth as a whole, and pull it back towards the book and the tractor. For the blue arrows — the normal forces from the table which push the book and the tractor up and prevent them from falling down — the corresponding reaction forces act on the table and push it down. Finally, the reaction forces for the friction forces on the book and the tractor (purple arrows) also act on the table, but in the horizontal directions: the friction force from the book $f_{\text{table}}^{(\text{book})}$ pushes the table to the right, while the friction force from the tractor $f_{\text{table}}^{(\text{tractor})}$ pushes the table to the left.

Textbook problem **SP6** at the end of chapter **4**:

(a) The true weight of a body is the gravitational force $W = mg$ acting on it. For the man in question, his true weight is

$$W \;=\; mg \;=\; 60\text{ kg} \times 9.8\text{ N/kg} \;=\; 588\text{ N} \;\approx\; 590\text{ N}. \tag{8}$$

Or in American units, about 130 pounds.

Note accuracy: I assume the man's mass is accurate to the nearest kilogram since that's the usual accuracy of the bathroom scales, thus $m = 60 \pm 0.5$ kg. If the mass is less accurate — *i.e.*, $m = 60 \pm 5$ kg, then all the answers in this problem should be rounded off to a single significant figure.

(b) By the Second Law of Newton, the net force acting on the body is related to its acceleration as $m\vec{\mathbf{a}} = \vec{\mathbf{F}}^{\text{net}}$. The man standing in the elevator has the same acceleration as the elevator itself, namely $a_x = 0$, $a_y = -1.4\text{ m/s}^2$ (the elevator is accelerating down). Hence, the net force acting on the man is

$$\begin{aligned} F_x^{\text{net}} &= ma_x \;=\; 0, \\ F_y^{\text{net}} &= ma_y \;=\; (60\text{ kg}) \times (-1.4\text{ m/s}) \;=\; -84\text{ N}. \end{aligned} \tag{9}$$

In other words, the net force has magnitude 84 N and downward direction.

(c) There are two forces acting on the man, its weight $W = mg$ and the normal force N from the elevator's floor. Both of these forces are vertical — which explains $F_x^{\rm net} = 0$ — but N points up while $W = mg$ points down, hence

$$F_y^{\rm net} \;=\; +N \;-\; W. \tag{10}$$

Given the weight W from part (a) and the net force from part (b), the normal force from the floor must be

$$N \;=\; F_y^{\rm net} \;+\; W \;=\; -84\ {\rm N} \;+\; 588\ {\rm N} \;=\; 504\ {\rm N} \;\approx\; 5\underline{0}0\ {\rm N}. \tag{11}$$

(d) Bathroom scales measure the downward normal force from the body standing on them. By the Third Law of Newton, this is also the upward normal force from the scales acting on the body itself.

If a man stands on the scales in a moving elevator, he accelerates with the elevator just as if he stood directly on the elevator floor, so the net force on him should be exactly the same as in part (b). Consequently, the normal force on him should be exactly the same as in part (c), namely $N \approx$ 500 N. And that's what the scales would measure as his apparent weight — 84 N lighter than his true weight mg.

Of course, most bathroom scales in this country report the apparent weight — *i.e.*, the normal force they produce — in pounds rather than in Newtons, so what they will show is about 110 pounds, 20 pounds lighter than the man's true weight.

(e) If the elevator — and the man inside it — accelerate up rather than down, $a_y = +1.4\ {\rm m/s}^2$, then the net force acting on the man should be

$$\begin{aligned} F_x^{\rm net} \;&=\; ma_x \;=\; 0, \\ F_y^{\rm net} \;&=\; ma_y \;=\; (60\ {\rm kg}) \times (+1.4\ {\rm m/s}) \;=\; +84\ {\rm N}. \end{aligned} \tag{12}$$

Since his true weight mg remains the same, the normal force from the elevator's floor — or from the bathroom scales — on the man must be

$$N \;=\; F_y^{\rm net} \;+\; W \;=\; +84\ {\rm N} \;+\; 588\ {\rm N} \;= 672\ {\rm N} \approx 670\ {\rm N}. \tag{13}$$

And if he stands on the scales, than 670 N is what they will show as his apparent weight — 84 N heavier than his true weight.

On in American units, the scales would show about 150 pounds, 20 pounds heavier than the man's true weight.

<u>The bottom line:</u>

In an elevator accelerating down, the apparent weight of a body is smaller than its true weight. In an elevator accelerating up, the apparent weight of a body is larger than its true weight.

<u>Textbook problem **SP5** at the end of chapter **4**</u>:

(a) The string tension T pulls the left block to the right and also pulls the right block to the left. Totaling the horizontal forces acting on each block, we have

$$F_x^{\text{net on left}} \;=\; T \;-\; 6\ {\rm N}, \tag{14}$$

$$F_x^{\text{net on right}} \;=\; -T \;-\; 8\ {\rm N} \;+\; 3\underline{0}\ {\rm N}. \tag{15}$$

The net force acting on the two-block system is the sum of these two forces,

$$F_x^{\text{net on both}} \;=\; F_x^{\text{net on left}} \;+\; F_x^{\text{net on right}} \;=\; -6\ {\rm N} \;-\; 8\ {\rm N} \;+\; 3\underline{0}\ {\rm N} \;=\; +16\ {\rm N}. \tag{16}$$

Note that the string tension T cancels out of this net force. This is an example of a general rule: The net force acting on a many-body system is the sum of *external* forces acting on all the bodies, but the internal forces between the bodies do not contribute — by the Third Law of Newton, they always cancel out.

(b) Newton's Second Law can be applied to individual bodies or to a systems of bodies traveling together — like the two blocks in question. The second-law equation for the two-block system is

$$m^{\rm net}\vec{\mathbf{a}} \;=\; \vec{\mathbf{F}}^{\rm net\,on\,both}, \tag{17}$$

where $m^{\rm net}$ is the net mass of the two blocks,

$$m^{\rm net} \;=\; m^{\rm left} \;+\; m^{\rm right} \;=\; 2\ {\rm kg} \;+\; 4\ {\rm kg} \;=\; 6\ {\rm kg}. \tag{18}$$

Thus, given the net horizontal force found in part (a), the horizontal acceleration of the system is

$$a_x \;=\; \frac{F_x^{\rm net\,on\,both}}{m^{\rm net}} \;=\; \frac{+16\ {\rm N}}{6\ {\rm kg}} \;\approx\; 2.7\ {\rm m/s}^2. \tag{19}$$

(and the vertical acceleration is obviously zero, since both blocks are laying on a horizontal table.)

(c) Now let's apply the Second Law to the left block:

$$m^{\rm left}\,a_x \;=\; F_x^{\rm net\,on\,left} \;=\; T \;-\; 6\ {\rm N}, \tag{20}$$

where the second equality is eq. (14) for the net force on the left block. Given the acceleration found in part (b), we may solve this equation for the unknown string tension T:

$$T \;=\; 6\ {\rm N} \;+\; m^{\rm left}\,a_x \;=\; 6\ {\rm N} \;+\; 2\ {\rm kg}\times 2.7\ {\rm m/s}^2 \;\approx\; 11\ {\rm N}. \tag{21}$$

(d) Given the string tension T we have just found, the net force on the right block is

$$F_x^{\rm net\,on\,right} \;=\; -(T=11\ {\rm N}) \;-\; 8\ {\rm N} \;+\; 3\underline{0}\ {\rm N} \;=\; +11\ {\rm N}. \tag{22}$$

Consequently, by the Second Law applied to the right block, its acceleration is

$$a_x \;=\; \frac{F_x^{\rm net\,on\,right}}{m^{\rm right}} \;=\; \frac{+11\ {\rm N}}{4\ {\rm kg}} \;\approx\; +2.7\ {\rm m/s}^2, \tag{23}$$

the same as the system's acceleration obtained in part (b).

PS: Because of round-off errors, you may get slightly different numbers in parts (b) and (d), for example 2.7 m/s^2 for the two-block system in part (b) and 2.8 m/s^2 for the right block in part (d). In reality, the two accelerations should be exactly the same.

The best way to avoid round-off errors is to keep extra figures during all intermediate calculations, and only at the end round off all the answers. In this way, in part (b) we should get

$$a_x \;=\; \frac{16 \text{ N}}{6 \text{ kg}} = 2.667 \text{ m/s}^2 \tag{24}$$

(which at the end should be rounded off to 2.7 m/s^2, but only at the end), hence in part (c)

$$T \;=\; 6 \text{ N} \;+\; 2\text{kg} \times 2.667 \text{ m/s}^2 \;=\; 11.333 \text{ N} \tag{25}$$

(which should be rounded off to 11 N, but only at the end), and finally in part (d)

$$F_x^{\text{net on right}} \;=\; -11.333 \text{ N} \;-\; 8 \text{ N} \;+\; 30 \text{ N} \;=\; +10.667 \text{ N} \tag{26}$$

and hence

$$a_x \;=\; \frac{+10,667 \text{ N}}{4 \text{ kg}} \;=\; +2.667 \text{ m/s}^2, \tag{27}$$

exactly as in part (b).