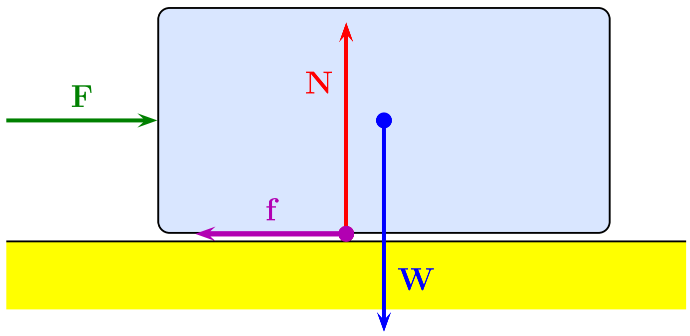
Non-textbook problem #**I**:

There are 4 forces acting on the box: Its own weight $\mathbf{W} = m\mathbf{g}$, the normal force $\mathbf{N}$ from the floor, the friction force $\mathbf{f}$ between the floor and the box, and the pushing force $\mathbf{F}$. The diagram below shows their directions:



By the Second Law of Newton,

$$ma_x = F_x^{\text{net}} = F - f, \tag{1}$$
$$ma_y = F_y^{\text{net}} = N - W, \tag{2}$$

and since the box never moves vertically, eq. (2) implies

$$N - W = ma_y = 0 \tag{3}$$

and hence

$$N = W = mg = 68\text{ kg} \times 9.8\text{ N/kg} = 666\text{ N} \approx 670\text{ N}. \tag{4}$$

As to the horizontal motion, at first the box does not move, which means

$$F - f = ma_x = 0 \implies f = F. \tag{5}$$

During this time, the friction force $\mathbf{f}$ is the *static friction* force: It adjusts its direction and magnitude to whatever it takes to prevent the motion, in our case the magnitude is $f = F$

while the direction opposes the pushing force. However, the magnitude of the static friction force is limited by the normal force,

$$f_s \leq f_s^{\max} = \mu_s \times N \tag{6}$$

where μ_s is the static friction coefficient. This means that as long as the pushing force F does not exceed this limit, the static friction force cancels the pushing force and the box remains at rest. But once the pushing force exceeds the limit (6), the static friction force gives up and the box begins to move. For the box in question, this happens for $F^{\max} = 330$ N, which means that the maximal static friction force on the box is is

$$f_s^{\max} = \mu_s \times N = 330 \text{ N}. \tag{7}$$

Comparing this value to the normal force (4), we find the *static friction coefficient*

$$\mu_s = \frac{f^{\max}}{N} = \frac{330 \text{ N}}{666 \text{ N}} = 0.495 \approx 0.50. \tag{8}$$

Once the box begins to slide, the static friction force turns into a *kinetic friction* force whose magnitude is completely determined by the normal force,

$$f_k = \mu_k \times N, \tag{9}$$

where the kinetic friction coefficient μ_k is less than the static friction coefficient μ_s, and that's why the kinetic friction force is less than the maximal static friction force.

For the moving box in question, we may find the value of the kinetic friction force from the Second Law equation (1):

$$f_k = F - ma_x = 330 \text{ N} - 68 \text{ kg} \times 1.0 \text{ m/s}^2 = 262 \text{ N} \approx 260 \text{ N}. \tag{10}$$

Plugging this kinetic friction force and the normal force (4) into eq. (9), we find the *kinetic friction coefficient*

$$\mu_k = \frac{f_k}{N} = \frac{262 \text{ N}}{666 \text{ N}} = 0.393 \approx 0.39. \tag{11}$$

Non-textbook problem #**II**:

For simplicity, let's neglect the air drag on the moving car and assume that the road is horizontal. Then the only horizontal force on the cat is the friction force f between the tires and the road, so the horizontal acceleration of the car is simply

$$a_x \;=\; \pm\frac{f}{m} \tag{12}$$

where the sign depends on the direction of the friction force: positive when you accelerate forward, and negative when you hit the break.

Assuming the friction force is constant during the breaking, the acceleration is negative and constant, and the stopping distance under such conditions is

$$L \;=\; \frac{v_0^2}{2|a|} \tag{13}$$

(see solutions to the first problem of the first midterm), or in terms of the friction force

$$L \;=\; \frac{m\times v_0^2}{2f}\,. \tag{14}$$

Clearly, the shortest possible stopping distance calls for the strongest possible friction force,

$$L^{\min} \;=\; \frac{m\times v_0^2}{2f_{\max}}\,. \tag{15}$$

(a) If the car does not skid, its tires roll on the road such that the bottom point of each tire has zero velocity relative to the road — the rotational velocity of the tire and the linear velocity of the car cancel each other. Consequently, the friction force f between the tires and the road is *static* rather then kinetic. The magnitude of this force is whatever it takes to prevent the skid — which depends on the rotational deceleration of the tires by the breaking system. That's how you control the rate of deceleration by adjusting the pressure on the break pedal! However, regardless of the breaks, there is a limit of the static friction force

$$f_s \;\leq\; \mu_s\times N \tag{16}$$

where μ_s is the static friction coefficient ($\mu_s = 0.40$ for the car in question) while N is the normal force of the road on the car.

To find the normal force, we note that the car does not accelerate vertically, so the vertical forces on it must vanish,

$$F_y^{\rm net} \;=\; ma_y \;=\; 0. \tag{17}$$

There are two vertical force on the car — its weight $W = mg$ and the normal force N, so they must cancel each other,

$$F_y^{\rm net} \;=\; N \;-\; mg \;=\; 0 \quad\Longrightarrow\quad N \;=\; mg. \tag{18}$$

Consequently, the maximal static friction force on the car is

$$f_s^{\rm max} \;=\; \mu_s \times mg. \tag{19}$$

Plugging this maximal force into eq. (15) for the shortest stopping distance, we find

$$L_{\rm no\,skid}^{\rm min} \;=\; \frac{m \times v_0^2}{2\mu_s \times mg} \;=\; \frac{v_0^2}{2\mu_s \times g}\,. \tag{20}$$

Note that the car's mass m cancels out from this formula.

Numerically, for the car in question, the shortest no-skidding stopping distance is

$$L_{\rm no\,skid}^{\rm min} \;=\; \frac{(22\ {\rm m/s})^2}{2 \times 0.40 \times 9.8\ {\rm m/s}^2} \;\approx\; 62\ {\rm m}, \tag{21}$$

about 200 feet.

(b) If you push the breaks too hard, the static friction force becomes unable to synchronize the linear motion of the car with the rotational motion of the tires. Instead, the wheels stop rotating, and the bottoms of the tires start sliding on the road. Consequently, the friction force between the tires and the road becomes kinetic rather than static, and its magnitude is no longer controlled by the break. Instead, it's completely determined by the normal force (18),

$$f_k \;=\; \mu_k \times N \;=\; \mu_k \times mg, \tag{22}$$

where μ_k is the kinetic friction coefficient. Since $\mu_k < \mu_s$, the kinetic friction force is weaker than the maximal static friction force, which makes for a longer stopping distance:

$$L_{\rm skid}^{\rm min} \;=\; \frac{m \times v_0^2}{2f_k} \;=\; \frac{m \times v_0^2}{2\mu_k \times mg} \;=\; \frac{v_0^2}{2\mu_k \times g} \tag{23}$$

— this linger stopping distance is also independent on the car's mass. Numerically, for the car in question,

$$L_{\rm skid}^{\rm min} \;=\; \frac{(22\ {\rm m/s})^2}{2 \times 0.30 \times 9.8\ {\rm m/s}^2} \;\approx\; 82\ {\rm m}, \tag{24}$$

about 270 feet.

<u>Non-textbook problem #**III**</u>:

The air drag on a parachute is proportional to the square of the falling speed,

$$F_d \;=\; D \times v^2. \tag{25}$$

As a parachute jumper accelerates downward, the drag force on the chute slows down the acceleration. Given enough falling time, the jumper reaches the terminal speed v_t at which the drag force cancels the jumper's weight mg and the acceleration ceases,

$$ma_y \;=\; F_d(v) \;-\; mg \;=\; 0 \quad \text{for } v = v_t. \tag{26}$$

This gives us a simple equation for the terminal speed,

$$F_d(v_t) \;=\; mg, \tag{27}$$

or in light of eq. (25) for the drag force,

$$D \times v_t^2 \;=\; mg. \tag{28}$$

Solving this equation gives us

$$v_t = \sqrt{\frac{mg}{D}}. \tag{29}$$

For the parachute and the jumper in question, the terminal speed is

$$v_t = \sqrt{\frac{85\ \text{kg} \times 9.8\ \text{m/s}^2}{52\ \text{kg/m}}} = \sqrt{16\ \text{m}^2/\text{s}^2} = 4.0\ \text{m/s}, \tag{30}$$

about 9 miles per hour.

<u>Non-textbook problem **#IV**</u>:

(a) Relative to the Earth's center, every locale fixed to the Earth surface moves in a latitude circle at a uniform speed: the whole circle in 24 hours. The radius of this circle depends on the geographic latitude,

$$R_{\text{circle}} = R_{\text{Earth}} \times \cos(\text{latitude}), \tag{31}$$

that's why Anchorage (Alaska) moves in a smaller circle that Austin (Texas). The length of the circle is $L = 2\pi R$, so the speed of the circular motion is

$$v = \frac{L = 2\pi R}{T = 24\ \text{hours}}. \tag{32}$$

In particular,

$$v_{\text{Austin}} = \frac{2\pi \times 5500\ \text{km}}{24\ \text{hours}} \approx 1440\ \text{km/hr} \approx \underline{40}0\ \text{m/s}, \tag{33}$$

while

$$v_{\text{Anchorage}} = \frac{2\pi \times 3100\ \text{km}}{24\ \text{hours}} \approx 810\ \text{km/hr} \approx \underline{22}5\ \text{m/s}. \tag{34}$$

(b) The centripetal acceleration of a body moving in circle of radius R at uniform speed v is

$$a_c \;=\; \frac{v^2}{R}\,. \tag{35}$$

In particular, the centripetal acceleration of Austin (Texas) is

$$a_c(\text{Austin}) \;=\; \frac{v^2_{\text{Austin}}}{R_{\text{Austin}}} \;=\; \frac{(400\ \text{m/s})^2}{5500\cdots 10^3\ \text{m}} \;\approx\; 2.9\cdot 10^{-2}\ \text{m/s}^2, \tag{36}$$

while the centripetal acceleration of Anchorage (Alaska) is

$$a_c(\text{Anchorage}) \;=\; \frac{v^2_{\text{Anchorage}}}{R_{\text{Anchorage}}} \;=\; \frac{(225\ \text{m/s})^2}{3100\cdots 10^3\ \text{m}} \;\approx\; 1.6\cdot 10^{-2}\ \text{m/s}^2. \tag{37}$$

Note: for each city, the direction of the centripetal acceleration is towards the center of the latitude circle rather than to the center of the spherical Earth. In the local coordinates (North, South, East, West, vertically up, and vertically down), the acceleration vector is tilted to the North from vertically down by the angle equal to the city's latitude.

(c) According to eq. (35), the centripetal acceleration should decrease with the circle's radius R. And yet, despite moving in a bigger circle, Austin has a larger centripetal acceleration than Anchorage. The solution to this paradox is very simple: Austin has a higher speed than Anchorage, which increases its acceleration by a bigger factor than the larger radius decreases it.

The best way to see how this works is to express the centripetal acceleration in terms of the radius R and the rotational period T instead of the radius and the speed v. In terms of the radius and the period, the speed is

$$v \;=\; \frac{L}{T} \;=\; \frac{2\pi R}{T}\,, \tag{38}$$

hence

$$v^2 \;=\; \frac{4\pi^2 R^2}{T^2} \tag{39}$$

and therefore, the centripetal acceleration is

$$a_c \;=\; \frac{v^2}{R} \;=\; \frac{4\pi^2 R^2}{T^2}\times\frac{1}{R} \;=\; \frac{4\pi^2 R}{T^2}\,. \tag{40}$$

We see that *for a fixed period T of rotation*, the centripetal acceleration *increases* with the radius! And since every place on Earth rotates with the same period $T = 24$ hours, it follows that places nearer the equator have higher acceleration than the places nearer to one of the poles. In particular, Anchorage (Alaska) has a higher acceleration than Austin (Texas).

The bottom line: For a fixed speed of motion, the centripetal acceleration is inversely proportional to the radius, but for a fixed period of rotation, the acceleration is directly proportional to the radius. To avoid confusion, it is better to remember formulae like

$$a_c = \frac{v^2}{R} = \frac{4\pi^2 R}{T^2} \tag{41}$$

than text passages with words 'directly proportional' or 'inversely proportional'. The proportionality may depend on what other factors are held fixed or allowed to change, while a formula contains all the factors in a compact form.

Non-textbook problem **#V**:

(a) The car moves along a circular arc of radius $R = 95$ m at speed $v = 22$ m/s, so its centripetal acceleration is

$$a_c = \frac{v^2}{R} = \frac{(22\text{ m/s})^2}{95\text{ m}} = 5.1\text{ m/s}^2. \tag{42}$$

(b) The center of the circle is below the hilltop, so when the car goes through the top, its centripetal acceleration is directed vertically down, $a_y = -a_c$ while $a_x = 0$. The vertical forces acting on the car are its weight mg (pulling down) and the normal force N (pushing up), thus

$$F_y^{\text{net}} = N - mg. \tag{43}$$

By the Second Law of Newton,

$$F_y^{\text{net}} = ma_y = -ma_c\,, \tag{44}$$

hence

$$N - mg = F_y^{\text{net}} = -ma_c \tag{45}$$

and consequently

$$N = mg - ma_c = m(g - a_c) = 800 \text{ kg} \times (9.8 \text{ m/s}^2 - 5.1 \text{ m/s}^2) \approx 3800 \text{ N}. \quad (46)$$

Note that this normal force is significantly smaller than the car's weight $mg \approx 7800$ N.

(c) If the other car were to follow the same path as the first car but at higher speed $v =$ 33 m/s. it would have a larger centripetal acceleration

$$a_c = \frac{v^2}{R} = \frac{(33 \text{ m/s})^2}{95 \text{ m}} = 11.5 \text{ m/s}^2. \quad (47)$$

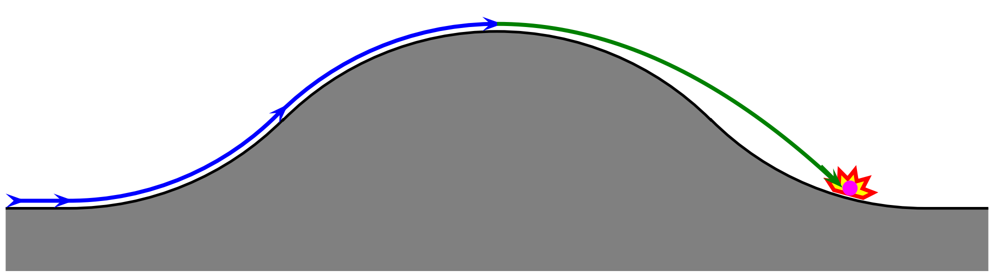
This acceleration is larger than the free-fall acceleration g, so the second car would need a negative (*i.e.*, downward) normal force! Indeed, proceeding similar to part (b) we would have

$$N = m(g - a_c) < 0. \quad (48)$$

However, the normal force between the road and the car's tires pushes but does not pull — for the car on top of the road, the direction of N is up but never down! Consequently, the car cannot have a downward acceleration larger than g, which means that it cannot stay on the road when

$$a_c = \frac{v^2}{R} > g. \quad (49)$$

Instead, the car will fly off the road, as shown on the picture below:



Non-textbook problem #**VI**:

There are three forces acting on the car, its weight mg, the normal force N from the road, and the friction force f between the road and the tires. On a flat horizontal road, the normal force is vertical while the friction force is horizontal, hence

$$N - mg = ma_{\text{vertical}} = 0 \quad\Longrightarrow\quad N = mg \tag{50}$$

while the friction force provides the horizontal acceleration — speeding up, or slowing down, or turning right or left,

$$ma_{\text{hor}} = f. \tag{51}$$

This friction force — whether static or kinetic — does not get stronger than the maximal static friction

$$f_s^{\max} = \mu_s \times N = \mu_s \times mg, \tag{52}$$

so the horizontal acceleration cannot be larger than

$$a_{\text{hor}}^{\max} = \frac{f^{\max}}{m} = \mu_s \times g. \tag{53}$$

Suppose the car moves at constant speed v along a curve of radius R_c. The normal (sideways) acceleration of the car due to turning is

$$a_n = \frac{v^2}{R}. \tag{54}$$

In light of the acceleration limit (53), the car must have

$$\frac{v^2}{R} \le \mu_s \times g. \tag{55}$$

For a given car speed, this gives us a lower limit on the radius of curvature,

$$R_c \ge \frac{v^2}{\mu_s \times g} \tag{56}$$

and if the road curves sharper than this, $R_{\text{road}} < R_c$, then the car would not be able to take this curve — it would skid to a side and end up in a ditch or worse!

In order to follow the road, the car needs

$$\frac{v^2}{R_{\text{road}}} \ \le\ \mu_s g \quad\Longrightarrow\quad v^2 \ \le\ \mu_s g \times R_{\text{road}}\,, \tag{57}$$

which means it should not move faster than

$$v^{\max} \ =\ \sqrt{\mu_s g \times R_{\text{road}}}\,. \tag{58}$$

For the car and road in question,

$$v^{\max} \ =\ \sqrt{0.6 \times 9.8\ \text{m/s}^2 \times 30\ \text{m}} \ =\ 13.3\ \text{m/s} \ \approx\ 30\ \text{MPH}. \tag{59}$$

If the car moves at a slower speed, it would be able to follow the curving road, but if it goes faster than 30 MPH, it would end up in a ditch.

PS: What if the car's speed is not constant while it goes through the curve? In this case, the car's acceleration has two components, the normal acceleration a_n due to turning, and also the tangential acceleration a_t due to speeding up or slowing down. The tangential acceleration points forward or backward, while the normal acceleration is $\perp$ to motion, so the net horizontal acceleration is

$$a_{\text{hor}} \ =\ \sqrt{a_n^2 + a_t^2}\,. \tag{60}$$

All this acceleration is provided by the friction force, which limits it to

$$\sqrt{a_n^2 + a_t^2} \ \le\ \mu_s g. \tag{61}$$

For the normal acceleration, this gives us a stricter limit

$$a_n \ =\ \frac{v^2}{R} \ \le\ \sqrt{(\mu_s g)^2 - a_t^2} \ <\ \mu_s g \tag{62}$$

and hence a stricter speed limit then (58).

Non-textbook problem #**VII**:

When the velocity vector of a body changes both its magnitude (the speed) and its direction, the acceleration vector of the body has two components: The *tangential acceleration*

$$a_t = \frac{\Delta|\vec{\mathbf{v}}|}{\Delta t} \tag{63}$$

due to changing speed, and the *normal acceleration*

$$a_n = \frac{v^2}{R_{\text{curvature}}} \tag{64}$$

due to changing direction. As vectors, the tangential acceleration $\vec{\mathbf{a}}_t$ is parallel to the velocity vector $\vec{\mathbf{v}}$ (pointing forward when the speed increases and backward when the speed decreases) while the normal acceleration $\vec{\mathbf{a}}_n$ is perpendicular to the velocity vector $\vec{\mathbf{v}}$. In terms of the body's trajectory, the $\vec{\mathbf{a}}_t$ points along the tangent while the $\vec{\mathbf{a}}_n$ points normally (*i.e.*, perpendicularly) to the tangent, hence their name.

The net acceleration vector of the body is

$$\vec{\mathbf{a}} = \vec{\mathbf{a}}_t + \vec{\mathbf{a}}_n\,, \tag{65}$$

and since the two components are $\perp$ to each other, the magnitude of the net acceleration is

$$|\vec{\mathbf{a}}| = \sqrt{a_t^2 + a_n^2}\,. \tag{66}$$

The train in question has its speed increasing from $v_0 = 20$ m/s to $v' = 40$ m/s in Δt=20 s *at a uniform rate*, so its *tangential acceleration* during this time is constant

$$a_t = \frac{\Delta v = v' - v_0}{\Delta t} = \frac{40\text{ m/s} - 20\text{ m/s}}{20\text{ s}} = 1.0\text{ m/s}^2. \tag{67}$$

At the moment the train's speed was $v = 30$ m/s, its normal acceleration was

$$a_n = \frac{v^2}{R} = \frac{(30\text{ m/s})^2}{1200\text{ m}} = 0.75\text{ m/s}^2,. \tag{68}$$

and the net acceleration of the train at that moment had magnitude

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(1.0\text{ m/s}^2)^2 + (0.75\text{ m/s}^2)^2} = 1.25\text{ m/s}^2. \tag{69}$$

Note: the train moves in the horizontal plane, so its normal acceleration and the tangential acceleration are both horizontal, and the net acceleration vector is horizontal.

The net horizontal force on a train car follows from the net horizontal acceleration via the Second Law of Newton,

$$F = ma = 60\,000 \text{ kg} \times 1.25 \text{ m/s}^2 = 75\,000 \text{ N}, \tag{70}$$

or about 17,000 pounds.

Textbook problem **SP2** from chapter 5:

(a) In one rotational period $t = 8.0$ s, a rider of the Ferris wheel travels along a complete circle of length $L = 2\pi R = 2\pi \times 12.0 \text{ m} = 75.4 \text{ m} \approx 75$ m, so his/her speed is

$$v = \frac{2\pi R}{t} = \frac{75.4 \text{ m}}{8.0 \text{ s}} = 9.42 \text{ m/s} \approx 9.4 \text{ m/s} \approx 21 \text{ MPH}. \tag{71}$$

(b) The centipetal acceleration of a rider is

$$a_c = \frac{v^2}{R} = \frac{(9.42 \text{ m/s})^2}{12.0 \text{ m}} \approx 7.4 \text{ m/s}^2. \tag{72}$$

(c–d) The net force acting on the rider moving with centripetal acceleration a_c must be

$$F^{\text{net}} = ma_c = 4\underline{0} \text{ kg} \times 7.4 \text{ m/s}^2 = 296 \text{ N} \approx 3\underline{0}0 \text{ N} \approx 67 \text{ pounds}, \tag{73}$$

and its direction should be towards the circle, *i.e.*towards the axis of the Ferris wheel. In particular, when the rider goes through the top of this circle, the net force should be directed down.

Physically, there are two vertical forces acting on the rider, his weight $W = mg$ and the normal force N from his seat, hence

$$F^{\text{net}}_{\text{down}} = -F^{\text{net}}_y = mg - N. \tag{74}$$

By itself, the gravity force $mg = 4\underline{0} \text{ kg} \times 9.8 \text{ m/s}^2 = 392 \text{ N} \approx 390$ N is stronger than the net force (73), so it is more then enough to provide for the rider's centripetal acceleration. (This is the answer to part (c).)

The excess of gravity force over the required net downward force is canceled by the normal force N according to eq. (72). Thus,

$$N \;=\; mg \;-\; F^{\rm net}_{\rm down} \;\approx\; 390\ {\rm N} \;-\; 3\underline{0}0\ {\rm N} \;=\; 90\ {\rm N}. \tag{75}$$

Note: We can get better accuracy by working the algebra before the arithmetics,

$$\begin{aligned} N \;&=\; W \;-\; F^{\rm net}_{\rm down} \;=\; mg \;-\; ma_c \;=\; m(g-a-c) \\ &=\; 4\underline{0}\ {\rm kg}\times(9.8\ {\rm m/s^2} \;-\; 7.4\ {\rm m/s^2}) =\; 4\underline{0}\ {\rm kg}\times 2.4\ {\rm m/s^2} \;=\; 96\ {\rm N}. \end{aligned} \tag{76}$$

(e) The normal force pushes but does not pull. Assuming the rider sits on top of his seat (and is not glued to it), the normal force can push him up but cannot pull him down, $N_y \geq 0$. Therefore, the vertical acceleration of the rider is limited by

$$ma_y \;=\; F^{\rm net}_y \;=\; N_y \;-\; mg \;\geq\; -mg \quad\Longrightarrow\quad a_y \;\geq\; -g. \tag{77}$$

In other words, the rider cannot have a downward acceleration greater than g.

If the Ferris wheel were to spin so fast that a rider's seat has a centripetal acceleration a_c larger than g, then at the top of the spin, the rider would be unable to follow his/her seat on its way down. Instead, he/she would feel the seat being yanked down from under his/her butt, and then the unfortunate rider would fly like a projectile until something stops his/her flight.

If rider is wearing a seat belt, it would pull the him/her down back into the seat. If there is no belt but the rider is inside a closed cabin, he/she would hit the ceiling with his/her head (ouch!). And if there is no ceiling, the rider would fly out from the Ferris wheel and hit the ground (double ouch!).