

Non-textbook problem #I:

The kinetic energy of a body depends on its mass and speed as

$$K = \frac{1}{2}mv^2 \quad (1)$$

Therefore, two bodies of respective masses m_1 and m_2 and speeds v_1 and v_2 have same kinetic energies whenever

$$\frac{1}{2}m_1v_1^2 = \frac{1}{2}m_2v_2^2 \quad (2)$$

Given the masses of the two balls in question and the speed of the bowling ball, we can solve this equation for the speed of the tennis ball as

$$\begin{aligned} v_2^2 &= \frac{m_1}{m_2} \times v_1^2, \\ v_2 &= \sqrt{\frac{m_1}{m_2}} \times v_1 = \sqrt{\frac{7.0 \cdot 10^3 \text{ g}}{57 \text{ g}}} \times 5.0 \text{ m/s} \approx 55 \text{ m/s}, \end{aligned} \quad (3)$$

or almost 124 miles per hour.

This speed is rather high for a tennis ball, but not impossibly so. [The best tennis players in the world sometimes serve the ball at this speed, or even faster.](#)

Non-textbook problem #II:

The bullet hits the tree with a kinetic energy

$$K_0 = \frac{1}{2}mv^2 = \frac{1}{2}(0.0090 \text{ kg})(250 \text{ m/s})^2 \approx 280 \text{ J}. \quad (4)$$

When the bullet penetrates the wood, there is a resisting force F opposing its motion. As the bullet moves through distance x inside the wood, this force performs negative work

$$W = x \times F_x = -xF, \quad (5)$$

where the minus sign comes from the opposite directions of the force F and the bullet's

displacement x . By the work energy theorem, the bullet's kinetic energy becomes

$$K = K_0 + W = K_0 - Fx, \quad (6)$$

and when this kinetic energy drops to zero, the bullet stops. Thus, at the stopping point x_s

$$K = K_0 - F \times x_s = 0, \quad (7)$$

so given the initial kinetic energy (4) and the stopping distance x_s , we may find the resisting force F as

$$F = \frac{K_0}{x_s} = \frac{280 \text{ J}}{0.080 \text{ m}} = 3500 \text{ N}, \quad (8)$$

about 800 pounds.

Non-textbook problem #III:

(a) There are three forces acting on the car: its weight $m\vec{g}$, the normal force \vec{N} from the road, and the kinetic friction force \vec{f}^* . The direction of the normal force \vec{N} is perpendicular to the road and hence to the car's displacement vector \vec{D} . Consequently, regardless of the normal force's magnitude, its mechanical work is zero,

$$W(\vec{N}) = \vec{D} \cdot \vec{N} = |D| \times |N| \times \cos(90^\circ) = 0. \quad (9)$$

The kinetic friction force \vec{f} has exactly opposite direction from the car's displacement \vec{D} , so it does negative work

$$W(\vec{f}) = |D| \times |f| \times \cos(180^\circ) = 120 \text{ m} \times 2500 \text{ N} \times (-1) = -3.0 \cdot 10^5 \text{ J}. \quad (10)$$

Finally, the gravity force $m\vec{g}$ pulls vertically down while the car slides in the direction 20° below the horizontal. Consequently, the angle between the gravity force and the car's displacement vector is $\theta = 90^\circ - 20^\circ = 70^\circ$, hence the work of the gravity force is

$$W(m\vec{g}) = |D| \times |mg| \times \cos(70^\circ) = 120 \text{ m} \times (1200 \text{ kg} \times 9.8 \text{ N/kg}) \times (+0.342) = +4.8 \cdot 10^5 \text{ J}. \quad (11)$$

* In principle, once the car starts moving fast, there is also the air drag force resisting its motion. But the air drag is so much weaker than the other three forces acting on the car that we are going to neglect it here.

(b) While the car slides 120 meters down the hill, the forces acting on it perform net work

$$W_{\text{net}} = W(\vec{\mathbf{N}}) + W(\vec{\mathbf{f}}) + W(m\vec{\mathbf{g}}) = 0 - 3.0 \cdot 10^5 \text{ J} + 4.8 \cdot 10^5 \text{ J} = +180\,000 \text{ J}. \quad (12)$$

By the work-energy theorem, the car's kinetic energy must increase by the same amount,

$$\Delta K \equiv K - K_0 = W_{\text{net}}, \quad (13)$$

and since the car starts with zero kinetic energy — it was at rest — it ends up with kinetic energy

$$K = K_0 + W_{\text{net}} = 0 + 180\,000 \text{ J} = 180\,000 \text{ J}. \quad (14)$$

This kinetic energy is related to the mass and speed of the car as

$$K = \frac{1}{2}mv^2, \quad (15)$$

so given the mass and the kinetic energy of the car, we may find its speed as

$$v^2 = \frac{2K}{m} \implies v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2 \times 180\,000 \text{ J}}{1200 \text{ kg}}} = 17.3 \text{ m/s} \approx 17 \text{ m/s}, \quad (16)$$

about 39 MPH.

Non-textbook problem #IV:

(a) Any mass m — regardless if it belongs to a solid body or just to some volume of water — in the gravitational field of Earth has potential energy

$$U = mgy \quad (17)$$

where y is the vertical coordinate — the elevation — of that mass m . As the water in Lake Mead flows through the turbines of the Hoover Dam and ends up at a lower elevation below

the dam, its potential energy decreases by

$$-\Delta U = mg \times (-\Delta y). \quad (18)$$

Every second, 1100 cubic meters (1.1 million liters or 291 thousand gallons) of water flow down through the turbines; this volume of water has mass

$$m = V \times \rho_{\text{water}} = 1100 \text{ m}^3 \times 1000 \text{ kg/m}^3 = 1.1 \cdot 10^6 \text{ kg}. \quad (19)$$

As this mass moves down by $-\Delta y = 180 \text{ m}$, its potential energy decreases by

$$-\Delta U = mg \times (-\Delta y) = 1.1 \cdot 10^6 \text{ kg} \times 9.8 \text{ N/kg} \times 180 \text{ m} = 1.94 \cdot 10^9 \text{ J} \approx 1.9 \cdot 10^9 \text{ J}. \quad (20)$$

In other words, the water flowing through the turbines of the Hoover Dam *releases* 1.9 billion Joules of potential energy every second.

(b) As the water releases its potential energy, it performs mechanical work spinning the turbines, which in turn perform mechanical work rotating the generators that produce the electricity. Altogether, 90% of the released potential energy of water turns into electric energy; the remaining 10% are lost as heat or the kinetic energy of water in the river below the Hoover Dam. Thus, every second the generators at the Hoover Dam produce

$$E = 0.90 \times 1.94 \cdot 10^9 \text{ J} = 1.75 \cdot 10^9 \text{ J} \quad (21)$$

of electric energy. In terms of the electric *power*, 1 Joule every second means 1 Watt, so the electric power generated by the Hoover Dam is

$$P = \frac{E}{t} = \frac{1.75 \cdot 10^9 \text{ J}}{1 \text{ second}} = 1.75 \cdot 10^9 \text{ W}$$

or 1.75 GigaWatt.

Non-textbook problem #V:

(a) The lowest point of the ball's path is directly below the suspension point of the cable. Let y be the vertical position of the ball *relative to the lowest point* of its trajectory while \tilde{y} is the vertical position of the same ball *relative to the suspension point of the cable*. Then

$$y \equiv \tilde{y} + L, \quad \tilde{y}_{\text{bot}} = -L, \quad y_{\text{bot}} = 0, \quad (22)$$

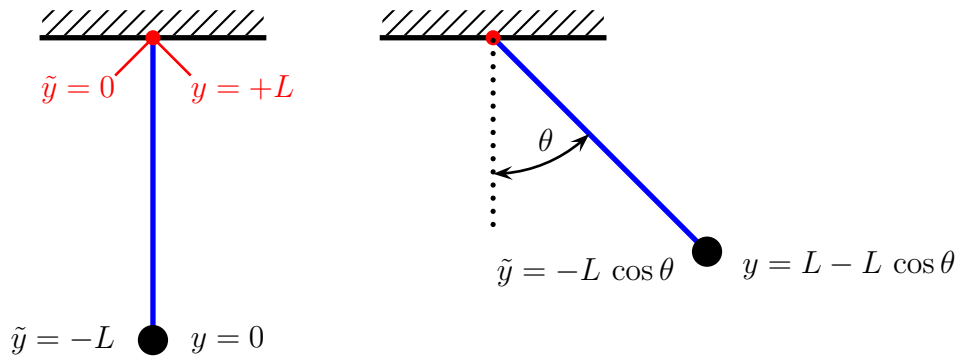
where L is the length of the cable. When the ball is swinging left or right and the cable makes angle θ with the vertical, the ball's vertical position relative to the suspension point is

$$\tilde{y} = L \times (-\cos \theta) = -L \cos \theta, \quad (23)$$

so the ball's position relative to its lowest point is

$$y = L - L \cos \theta = L \times (1 - \cos \theta) > 0. \quad (24)$$

This geometry is illustrated on the following diagram:



In particular, at the two turning points of the ball, the string is at angles $\theta = \pm 45^\circ$, namely $+45^\circ$ at the right turning point and -45° at the left turning point. For these points,

$$\cos(\theta = \pm 45^\circ) = \frac{\sqrt{2}}{2} = 0.707\dots, \quad (25)$$

so the vertical coordinate of the ball relative to the bottom point is

$$y_{\text{turn}} = L \times (1 - \cos \theta) = 3.0 \text{ m} \times (1 - 0.707) = +0.88 \text{ m}. \quad (26)$$

(b) The only non-gravitational force acting on the ball is the cable's tension T , and since this tension always pulls the ball in the direction perpendicular to its motion, it does not do any mechanical work, $W(T) = 0$. Therefore, the net mechanical energy (kinetic+potential) of the ball is conserved,

$$E = K + U = \frac{1}{2}mv^2 + mgy = \text{const.} \quad (27)$$

In other words, this energy has the same value at any point of the balls swing.

To find this value, we note that at a turning point, the ball has zero speed and hence zero kinetic energy, thus

$$E = \frac{1}{2}mv_{\text{turn}}^2 + mgy_{\text{turn}} = 0 + mgy_{\text{turn}}. \quad (28)$$

Consequently, at any other point of the trajectory, the ball has the same energy

$$\frac{1}{2}mv^2 + mgy = E = mgy_{\text{turn}}, \quad (29)$$

which allows us to find its speed from the vertical position,

$$\begin{aligned} \frac{1}{2}mv^2 &= mgy_{\text{turn}} - mgy, \\ &\Downarrow \\ v^2 &= 2g(y_{\text{turn}} - y), \\ &\Downarrow \\ v &= \sqrt{2g(y_{\text{turn}} - y)}. \end{aligned} \quad (30)$$

In particular, at the lowest point $y = 0$ of the trajectory, the ball has speed

$$v_{\text{bot}} = \sqrt{2gy_{\text{turn}}} = \sqrt{2 \times (9.8 \text{ m/s}^2) \times (0.88 \text{ m})} \approx 4.1 \text{ m/s} \approx 9.3 \text{ MPH}. \quad (31)$$

(c) The ball swings back and forth along a circular arc of radius $R = L$, so it has a centripetal acceleration

$$a_c = \frac{v^2}{L}. \quad (32)$$

At the bottom point, this centripetal acceleration is directed vertically up, while the tangential acceleration — if any — is horizontal. In fact, there is no horizontal centripetal acceleration at this point because both forces acting on the ball — its weight mg and the cable tension T — are vertical when the ball is at its lowest point, thus

$$ma_x = F_x^{\text{net}} = 0. \quad (33)$$

As to the vertical acceleration, the Second Law of Newton tells us

$$ma_y = F_y^{\text{net}} = T - mg. \quad (34)$$

Equating the vertical acceleration with the centripetal acceleration (32), $a_y = +a_c$, we see that the cable tension must be

$$T = ma_c + mg = m(a_c + g). \quad (35)$$

Numerically, at the bottom point the ball has speed $v = 4.1$ m/s (see part (b)), so its centripetal acceleration is

$$a_c = \frac{v^2}{L} = \frac{(4.1 \text{ m/s})^2}{3.0 \text{ m}} = 5.75 \text{ m/s}^2. \quad (36)$$

Hence the cable tension is

$$T = 2.0 \text{ kg} \times (9.8 \text{ m/s}^2 + 5.75 \text{ m/s}^2) \approx 31 \text{ N}, \quad (37)$$

about 7 pounds of force.

(c') *Algebraic solution for the cable tension:*

Let's derive an algebraic formula for the cable tension (when the ball is at its lowest point) in terms of the maximal angle θ_{\max} of the cable at a turning point of the ball.

In part (a) we saw that the height of the ball at a turning point relative to the bottom point is related to the maximal angle as

$$y_{\text{turn}} = L \times (1 - \cos \theta_{\max}). \quad (38)$$

In part (b) we related this height difference into the speed of the ball at the bottom point,

$$v_{\text{bot}}^2 = 2g \times y_{\text{turn}}, \quad (39)$$

so combining these two formulae we immediately obtain

$$v_{\text{bot}}^2 = 2g \times L \times (1 - \cos \theta_{\max}). \quad (40)$$

Consequently, the centripetal acceleration of the ball at the bottom point is

$$a_c(@ \text{bottom}) = \frac{v_{\text{bot}}^2}{L} = 2g \times (1 - \cos \theta_{\max}). \quad (41)$$

Note that the cable length L drops out from this formula!

Finally, the cable tension when the ball is at the bottom point is

$$T = mg + ma_c = mg \times (1 + 2 - 2 \cos \theta_{\max}). \quad (42)$$

In particular, for $\theta_{\max} = 45^\circ$, the ratio of this tension to the ball's weight is

$$\frac{T}{mg} = 1 + 2 - 2 \cos(45^\circ) = 3 - \sqrt{2} \approx 1.586, \quad (43)$$

hence $T = 1.586 \times mg = 1.586 \times 19.6 \text{ N} \approx 31 \text{ N}$.

(d) When the cable makes angle 30° with the vertical, the height of the ball above its lowest point is

$$y = L(1 - \cos \theta) = 3.0 \text{ m} \times (1 - \cos(30^\circ)) = +0.40 \text{ m}. \quad (44)$$

In part (b) we used the energy conservation to relate the speed of the ball to its vertical coordinate y . Applying eq. (30) to the ball at height (44), we obtain its speed as

$$v = \sqrt{2g(y_{\text{turn}} - y)} = \sqrt{2(9.8 \text{ m/s}^2)(0.88 \text{ m} - 0.40 \text{ m})} \approx 3.1 \text{ m/s} \approx 6.9 \text{ MPH}. \quad (45)$$

Non-textbook problem #VI:

(a) Let us assume the bungee cord is perfectly elastic and neglect the air resistance. Under these assumptions, all forces are conservative and the net mechanical energy of the system — comprising the kinetic and gravitational potential energies of the ball and the elastic potential energy of the cord — is conserved,

$$E = K + U_{\text{grav}} + U_{\text{elastic}} = \frac{1}{2}mv^2 + mgy + \frac{1}{2}k(L - L_0)^2 = \text{const} \quad (46)$$

Here L is the cord's length; depending on the jumper's y height above the water and the bridge's height H ,

$$L = \left\{ \begin{array}{ll} H - y > L_0 & \text{if the cord is stretched,} \\ L_0 > H - y & \text{if the cord is folded,} \end{array} \right\} = \max(H - y, L_0). \quad (47)$$

Initially, the ball is at $y = H$ and has zero speed, while the cord is folded, $L = L_0$, so the initial net energy is

$$E_0 = 0 + mgH + 0. \quad (48)$$

The net mechanical energy is conserved, $E \equiv E_0$, thus *at all times during the jump*

$$\frac{1}{2}mv^2 + mgy + \frac{1}{2}k(L - L_0)^2 = mgH. \quad (49)$$

But the way the net energy splits between the kinetic, the gravitational, and the elastic energies changes during the bungee jump. As the ball goes down, its gravitational energy

decreases and turns into its kinetic energy and the elastic energy of the cord. At first, while $y > H - L_0$ the cord is folded and has no elastic energy, so all the potential energy released by the ball becomes its kinetic energy. But when the ball goes below $H - L_0$, the cord becomes stretched and its elastic energy starts to grow. Eventually, it grows faster than the ball's gravitational energy is released, so the kinetic energy has to decrease and the ball slows down. Finally, the balls runs out of kinetic energy altogether and stops at some altitude y_b — this is the lowest point of the jump. After that, the motion reverses directions as the cord's tension yanks the ball up.

(b) At the lowest point $y_b = 1$ m reached by the ball, its speed is zero and the kinetic energy vanishes. At the same point, the gravitational energy is $mg y_b$ while the elastic energy of the bungee cord follows from its length

$$L = H - y_b = 61 \text{ m} - 1 \text{ m} = 60 \text{ m}. \quad (50)$$

Plugging these data into the energy conservation formula (49), we obtain

$$0 + mg y_b + \frac{k}{2} \times (H - y_b - L_0)^2 = mgH. \quad (51)$$

The problem gives us all the data in this formula except the force constant k of the bungee cord, so we may treat it as an equation for k . To solve this equation, we first re-arrange it as

$$\frac{k}{2} \times (H - y_b - L_0)^2 = mgH - mg y_b = mg \times (H - y_b) \quad (52)$$

and then immediately obtain

$$k = \frac{2mg(H - y_b)}{(H - y_b - L_0)^2}. \quad (53)$$

Numerically, the bungee cord in question has force constant

$$k = \frac{2(122 \text{ kg})(9.8 \text{ m/s}^2) \times (61 \text{ m} - 1 \text{ m})}{(61 \text{ m} - 1 \text{ m} - 21 \text{ m})^2} = 94 \text{ N/m}. \quad (54)$$

(c) The energy conservation equation (49) can be used to find the kinetic energy of the ball — and hence its speed — at any point of the trajectory:

$$\frac{1}{2}mv^2 = mgH - mgy - \frac{1}{2}k(L - L_0)^2 \quad (55)$$

where the length L of the cord depends on y according to eq. (47) and its force constant k was found in part (b). Specifically, when the ball is 11 meters above the water, the cord has length

$$H = \max(H - y; L_0) = \max(61 \text{ m} - 11 \text{ m}; 21 \text{ m}) = 50 \text{ m} \quad (56)$$

and hence elastic energy

$$\frac{1}{2}k(L - L_0)^2 = \frac{1}{2}(94 \text{ N/m}) \times (50 \text{ m} - 21 \text{ m})^2 = 39\,700 \text{ J}. \quad (57)$$

At the same time, the ball's gravitational energy has dropped by

$$mgH - mgy = mg(H - y) = 122 \text{ kg} \times 9.8 \text{ N/kg} \times (61 \text{ m} - 11 \text{ m}) = 59\,800 \text{ J}. \quad (58)$$

According to eq. (55), this leaves the ball with kinetic energy

$$K \equiv \frac{1}{2}mv^2 = mg(H - y) - \frac{1}{2}k(L - L_0)^2 = 59\,800 \text{ J} - 39\,700 \text{ J} = 20\,100 \text{ J}. \quad (59)$$

Consequently, the speed of the ball at this point (11 m above the water) is

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2 \times 20\,100 \text{ J}}{122 \text{ kg}}} \approx 18 \text{ m/s}, \quad (60)$$

about 40 MPH.

Non-textbook problem #VII:

(a) The momentum

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}} \quad (61)$$

of a moving body is a vector quantity: its direction follows the velocity vector $\vec{\mathbf{v}}$. When the tennis ball in question bounces off the wall, it changes the direction of motion, so its momentum vector changes by the vector difference

$$\Delta\vec{\mathbf{p}} = m\vec{\mathbf{v}}' - m\vec{\mathbf{v}} = m(\vec{\mathbf{v}}' - \vec{\mathbf{v}}) \quad (62)$$

In components,

$$\begin{aligned} \Delta p_x &= m(v'_x - v_x) = (0.056 \text{ kg}) \times ((-32 \text{ m/s}) - (+38 \text{ m/s})) = -3.9 \text{ kg} \cdot \text{m/s}, \\ \Delta p_y &= m(v'_y - v_y) = (0.056 \text{ kg}) \times ((+2 \text{ m/s}) - (+2 \text{ m/s})) = 0. \end{aligned} \quad (63)$$

(b) By the impulse-momentum theorem,

$$\Delta\vec{\mathbf{p}} = \vec{\mathbf{I}}^{\text{net}}, \quad (64)$$

the momentum of a body changes by the net impulse of all the forces acting on it.

A constant force $\vec{\mathbf{F}}$ acting for time t has impulse

$$\vec{\mathbf{I}} = \vec{\mathbf{F}}t. \quad (65)$$

For variable forces, one has to split the time period into very short intervals Δt and sum the impulses over the intervals,

$$\vec{\mathbf{I}} = \lim_{\Delta t \rightarrow 0} \sum \vec{\mathbf{F}}\Delta t. \quad (66)$$

In calculus terms, the impulse is the integral of the force over time. However, in terms of the *average force* $\vec{\mathbf{F}}_{\text{avg}}$ during time t , the impulse is simply

$$\vec{\mathbf{I}} = t\vec{\mathbf{F}}_{\text{avg}}. \quad (67)$$

The tennis ball in question suffers one strong force from the wall it collides with, while all the other forces acting on it (such as its weight mg) are much weaker. Thus, over the

3.5 millisecond duration of the collision, the net impulse on the ball is just the impulse of the wall force,

$$\vec{\mathbf{I}}^{\text{net}} \approx \vec{\mathbf{I}}(\text{wall force}) = t\vec{\mathbf{F}}_{\text{avg}}^{\text{wall}}. \quad (68)$$

But we know the net impulse from the impulse-momentum theorem (64), thus

$$t\vec{\mathbf{F}}_{\text{avg}}^{\text{wall}} = \vec{\mathbf{I}}^{\text{net}} = \Delta\vec{\mathbf{p}}, \quad (69)$$

which allows us to calculate the average force from the wall on the tennis ball during the collision as

$$\vec{\mathbf{F}}_{\text{avg}}^{\text{wall}} = \frac{\Delta\vec{\mathbf{p}}}{t}. \quad (70)$$

In components,

$$\begin{aligned} F_{x,\text{avg}}^{\text{wall}} &= \frac{\Delta p_x}{t} = \frac{-3.9 \text{ kg} \cdot \text{m/s}}{3.5 \cdot 10^{-3} \text{ s}} = -1120 \text{ N}, \\ F_{y,\text{avg}}^{\text{wall}} &= \frac{\Delta p_y}{t} = \frac{0}{3.5 \cdot 10^{-3} \text{ s}} = 0. \end{aligned} \quad (71)$$

In other words, the average force from the wall on the tennis ball during the collision was pointing in the negative x direction — presumably \perp to the wall — and had magnitude 1120 Newtons (about 250 pounds).

Non-textbook problem #VIII (canceled):

If the gun is held loosely when it fires, the only strong force acting during the shooting is the gas pressure force between the gun and the bullet, while all the other forces on the gun+bullet system stay weak. Consequently, the net momentum of the system does not change during the shooting

$$\vec{\mathbf{P}}_{\text{net}} = M_{\text{gun}}\vec{\mathbf{v}}_{\text{gun}} + M_{\text{bullet}}\vec{\mathbf{v}}_{\text{bullet}} = \text{const.} \quad (72)$$

However, this net momentum is re-distributed between the gun and the bullet. Before the shooting, neither the gun nor the bullet were moving, so their net momentum was zero,

$$\vec{\mathbf{P}}_{\text{net}} = M_{\text{gun}}\vec{\mathbf{0}} + M_{\text{bullet}}\vec{\mathbf{0}} = \vec{\mathbf{0}}. \quad (73)$$

After the shooting, the bullet flies forward, the gun recoils back, but their net momentum

remains the same zero it was before the shooting,

$$M_{\text{gun}}\vec{v}_{\text{gun}} + M_{\text{bullet}}\vec{v}_{\text{bullet}} = \vec{P}_{\text{net}} = \vec{0}. \quad (74)$$

Given the masses of the gun and the bullet and the bullet's velocity, this vector formula can be used to calculate the recoil velocity of the gun:

$$M_{\text{gun}}\vec{v}_{\text{gun}} = -M_{\text{bullet}}\vec{v}_{\text{bullet}}, \quad (75)$$

hence

$$\vec{v}_{\text{gun}} = -\frac{M_{\text{bullet}}}{M_{\text{gun}}}\vec{v}_{\text{bullet}}. \quad (76)$$

The minus sign here gives the direction of the recoil — opposite from the bullet's direction. And the speed of the recoil is simply

$$v_{\text{gun}} = \frac{M_{\text{bullet}}}{M_{\text{gun}}} \times v_{\text{bullet}}. \quad (77)$$

For the Smith & Wesson 44 Magnum revolver in question, the recoil speed is

$$v_{\text{gun}} = \frac{13 \text{ g}}{1400 \text{ g}} \times 495 \text{ m/s} = 4.6 \text{ m/s}, \quad (78)$$

about 10 miles per hour.

PS: If you hold the gun loosely in your hand when you fire it, the gun will fly back with that speed. If the recoil direction lines up with your face (very likely), the gun will hit it in about 0.15 seconds ($L_{\text{arm}}/v_{\text{gun}} \approx 70 \text{ cm}/4.6 \text{ m/s} = 0.15 \text{ s}$), faster than you can get away from it, and you will not enjoy the experience! Moreover, a loosely held gun will twist in your hand while the bullet is still getting out, so you are unlikely to hit your intended target!

So if you feel like getting a gun — as many people do in Texas — make sure you know how to hold the gun when you shoot. Also, if your hands are not strong enough to contain the recoil of a powerful gun like 44 Magnum, get a smaller gun, or go without.