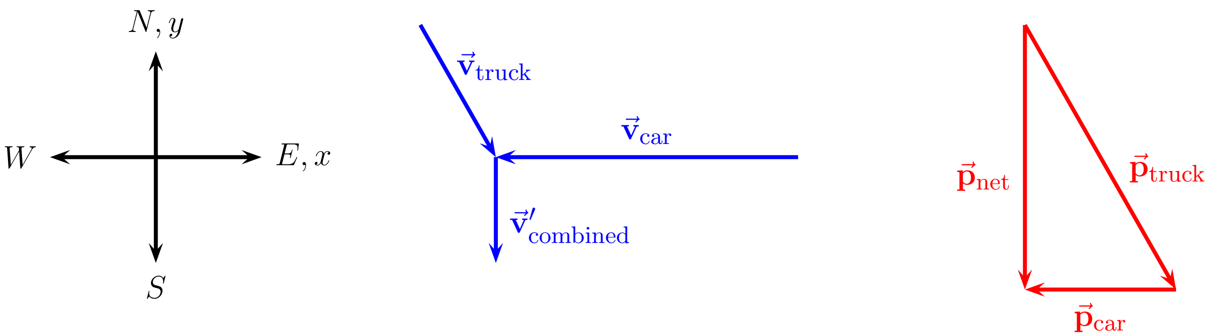
<u>Non-textbook problem #**I**</u>:



(1)

The collision in question is totally inelastic — after the collision, the two vehicles move together as one combined wreck with some common velocity $\vec{\mathbf{v}}'$. Consequently, their net momentum after the collision is

$$\vec{\mathbf{p}}'_{\text{net}} = m_c\vec{\mathbf{v}}' + m_t\vec{\mathbf{v}}' = (m_c + m_t)\vec{\mathbf{v}}'. \tag{2}$$

But the net momentum vector is conserved during the collision, so the net momentum of the car and the truck just before the collision was the same as the net momentum just after the collision, thus

$$\vec{\mathbf{p}}'_{\text{net}} = \vec{\mathbf{p}}_{\text{net}} = \vec{\mathbf{p}}_{\text{truck}} + \vec{\mathbf{p}}_{\text{car}}\,. \tag{3}$$

Graphically, the three momentum vectors $\vec{\mathbf{p}}'_{\text{net}}$, $\vec{\mathbf{p}}_{\text{truck}}$, and $\vec{\mathbf{p}}_{\text{car}}$ make a closed triangle, as shown on the right side of the diagram (1). Note that we know the directions of all three vectors: $\vec{\mathbf{p}}_{\text{truck}}$ points 30° East from due South because that was the truck's direction before the collision; likewise, $\vec{\mathbf{p}}_{\text{car}}$ points due West because that was the car's direction; finally, $\vec{\mathbf{p}}'_{\text{net}}$ points due South because that's where the combine wreckage went after the collision.

In (x, y) components, the vector equation (2) becomes

$$p'^{\,\text{net}}_x = p_{t,x} + p_{c,x}\,, \tag{4}$$

$$p'^{\,\text{net}}_y = p_{t,y} + p_{c,y}\,, \tag{5}$$

and for this problem, we need only the x components of the momenta.

(a) Before the collision, the truck had momentum of magnitude

$$p_t \;=\; m_t \times v_t \;=\; 4400\ \text{kg} \times 11\ \text{m/s} \;=\; 48\,400\ \text{N}\cdot\text{s}. \tag{6}$$

The direction of this momentum was 30° East from due South, so its Eastward component was

$$p_{t,x} \;=\; p_t \times \sin(30^\circ) \;=\; 48\,400\ \text{N}\cdot\text{s} \times 0.500 \;=\; 24\,200\ \text{N}\cdot\text{s}. \tag{7}$$

(b) After the collision, the combined wreck moves due South, so the net direction of the net momentum $\vec{\mathbf{p}}_{\text{net}} = \vec{\mathbf{p}}'_{\text{net}}$ is due South, and the x component of this net momentum vector is zero,

$$p_x^{\prime\,\text{net}} \;=\; 0. \tag{8}$$

(c) We may find the car's momentum before the collision — or at least its x component by solving eq. (4):

$$p_{c,x} \;=\; p_x^{\prime\,\text{net}} \;-\; p_{t,x} \;=\; 0 \;-\; 24\,200\ \text{N}\cdot\text{s} \;=\; -24\,200\ \text{N}\cdot\text{s}. \tag{9}$$

In other words, the car's momentum before the collision had a Westward component 24 200 Newton-seconds.

(d) The x component of the car's velocity can be immediately obtained from the x component of its momentum:

$$\vec{\mathbf{p}}_c \;=\; m_c \vec{\mathbf{v}}_c \quad\Longrightarrow\quad p_{c,x} \;=\; m_c \times v_{c,x}\,, \tag{10}$$

hence

$$v_{c,x} \;=\; \frac{p_{c,x}}{m_c} \;=\; \frac{-24\,200\ \text{N}\cdot\text{s}}{1100\ \text{kg}} \;=\; -22\ \text{m/s}. \tag{11}$$

Also, we know that before the collision, the car was moving due West, so the y component of its velocity was zero, $v_{c,y} = 0$. Consequently, the speed of the car was

$$v_c \;=\; \sqrt{v_{c,x}^2 + v_{c,y}^2} \;=\; |v_{c,x}| \quad \langle\!\langle\, \text{since } v_{c,y} = 0 \,\rangle\!\rangle \quad = \; 22\ \text{m/s}, \tag{12}$$

about 50 MPH. (Which is quite over the speed limit for the MLK boulevard.)

Non-textbook problem #**II**:

The collision between the two nuclei in question is a perfectly elastic head-on collision. My notes on collisions explain how to calculate the final velocities in such a collision. When both particles move before the collision, their velocities after the collision are given in eqs. (26–27) on page 5 of the notes:

$$\begin{aligned} v'_1 &= \frac{m_1 - m_2}{m_1 + m_2} \times v_1 \;+\; \frac{2m_2}{m_1 + m_2} \times v_2\,. \\ v'_2 &= \frac{m_2 - m_1}{m_1 + m_2} \times v_2 \;+\; \frac{2m_1}{m_1 + m_2} \times v_1\,. \end{aligned} \tag{13}$$

In our case, the particle #2 — the oxygen nucleus — is at rest before the collision, $v_2 = 0$, so eqs. (13) simplify to

$$\text{For } v_2 = 0, \quad v'_1 = \frac{m_1 - m_2}{m_1 + m_2} \times v_1\,, \quad v'_2 = \frac{2m_1}{m_1 + m_2} \times v_1\,, \tag{14}$$

cf. eqs. (29) and (31) on page 6 of my notes.

Since the oxygen nucleus is 4 times heavier than the helium nucleus (the alpha particle), $m_2 = 4 \times m_1$, the mass ratio in eq. (14) are

$$\begin{aligned} \frac{m_1 - m_2}{m_1 + m_2} &= \frac{1 - 4}{1 + 4} = -\frac{3}{5}, \\ \frac{2m_1}{m_1 + m_2} &= \frac{2 \times 1}{1 + 4} = +\frac{2}{5}. \end{aligned} \tag{15}$$

Hence, the two nuclei's velocities after the collisions are

$$\begin{aligned} v'_1 &= -\frac{3}{5} \times v_1 = -9.0 \cdot 10^6 \text{ m/s}, \\ v'_2 &= +\frac{2}{5} \times v_1 = +6.0 \cdot 10^6 \text{ m/s}. \end{aligned} \tag{16}$$

That is, the alpha particle bounces back (to where it came from) at speed 9 millions m/s, while the oxygen nucleus moves forward (in the original direction of the alpha particle) at speed 6 million m/s.

Non-textbook problem #**III**:

(a) To be precise, let the initial height $h = 1.2$ m be from the floor to the bottom of the basketball. That is, each ball falls down for 1.2 meters before colliding. Neglecting the air resistance to the balls' motion, we may determine each ball's speed from the energy conservation equation:

$$\tfrac{1}{2}mv^2 \;+\; mgy \;=\; E \;=\; \text{const} \;=\; \tfrac{1}{2}mv_0^2 \;+\; mgy_0\,. \tag{17}$$

Since each ball is dropped down with zero initial velocity, $v_0 = 0$, it ends up with

$$\tfrac{1}{2}mv^2 \;=\; mg(y_0 - y_{\text{fin}}) \;=\; mgh \quad\Longrightarrow\quad v^2 \;=\; 2gh. \tag{18}$$

Thus, just before the basketball hits the floor, its speed is

$$|v| \;=\; \sqrt{2gh} \;=\; \sqrt{2 \times 9.8\ \text{m/s}^2 \times 1.2\ \text{m}} \;=\; 4.85\ \text{m/s} \;\approx\; 4.8\ \text{m/s}, \tag{19}$$

that is, its *velocity* is $v_{\text{bb}} = -\sqrt{2gh} \approx -4.8$ m/s.

As to the tennis ball, it starts falling from the top of the basketball, and also ends its fall when it hits the basketball rather than the floor, so it has slightly higher y_0 and y_{fin} that the basketball, but the difference $y_0 - y_{\text{fin}} = h$ is exactly the same for the two balls. Consequently, just before the basketball hits the floor — which is just before the tennis ball itself hist the basketball — the tennis ball has the same velocity as the basketball,

$$v_{\text{tb}} \;=\; -\sqrt{2gh} \;\approx\; -4.8\ \text{m/s}. \tag{20}$$

(b) We assume a perfectly elastic head-on collision of the basketball with the floor. In such a collision, the relative velocity of the two bodies reverses direction but has the same magnitude before and after the collision. Since the floor is waay more massive then the basketball, its velocity after the collision is negligible, so to a very good approximation, the basketball simply reverses its velocity relative to the ground,

$$v'_{\rm bb} = -v_{\rm bb} = +\sqrt{2gh} \approx +4.8 \text{ m/s}. \tag{21}$$

To judge the quality of this approximation, me may use eq. (14) (*cf.* eq. (31) on page 6 of my notes on collisions) that takes the momentum conservation into account:

$$v'_{\rm bb} = \frac{m_{\rm bb} - m_{\rm floor}}{m_{\rm bb} + m_{\rm floor}} \times v_{\rm bb}\,. \tag{22}$$

Since the basketball weight less than a kilogram while the concrete floor weighs many tons, $m_{\rm floor} > 2000 \times m_{\rm bb}$, the mass ratio in this formula is very close to -1,

$$-1.000 < \frac{m_{\rm bb} - m_{\rm floor}}{m_{\rm bb} + m_{\rm floor}} < -0.999, \tag{23}$$

so indeed,

$$v_{\rm bb}^{\text{after bouncing from the floor}} \approx -v_{\rm bb}^{\text{before hitting the floor}} \tag{24}$$

as in eq. (21).

As to the tennis ball, its velocity does not noticeably change during a very short time it takes the basketball to bounce from the floor, so *just before the two balls collide with each other,*

$$v_{\rm tb} \approx -\sqrt{2gh} \approx -4.8 \text{ m/s}, \qquad v_{\rm bb} \approx +\sqrt{2gh} \approx +4.8 \text{ m/s}. \tag{25}$$

(c) Now consider the second collision — between the basketball that has bounced from the floor and the tennis ball still falling down. We assume this collision to be perfectly elastic and head-on, but now both balls are moving before the collision, so the velocities after the collision follow from the general eqs. (13), *cf.* eqs. (26–27) on page 5 of my notes:

$$\begin{aligned} v'_{\rm bb} &= \frac{m_{\rm bb} - m_{\rm tb}}{m_{\rm bb} + m_{\rm tb}} \times v_{\rm bb} + \frac{2m_{\rm tb}}{m_{\rm bb} + m_{\rm tb}} \times v_{\rm tb}\,. \\ v'_{\rm tb} &= \frac{m_{\rm tb} - m_{\rm bb}}{m_{\rm bb} + m_{\rm tb}} \times v_{\rm tb} + \frac{2m_{\rm bb}}{m_{\rm bb} + m_{\rm tb}} \times v_{\rm bb}\,. \end{aligned} \tag{26}$$

The velocities of the two balls just before the second collision are given by eq. (25). Plugging them into eqs. (26), we obtain:

$$\begin{aligned} v'_{\rm bb} &= \frac{m_{\rm bb}-m_{\rm tb}}{m_{\rm bb}+m_{\rm tb}}\times(+\sqrt{2gh}) + \frac{2m_{\rm tb}}{m_{\rm bb}+m_{\rm tb}}\times(-\sqrt{2gh}) = \frac{m_{\rm bb}-3m_{\rm tb}}{m_{\rm bb}+m_{\rm tb}}\times\sqrt{2gh},\\ v'_{\rm tb} &= \frac{m_{\rm tb}-m_{\rm bb}}{m_{\rm bb}+m_{\rm tb}}\times(-\sqrt{2gh}) + \frac{2m_{\rm bb}}{m_{\rm bb}+m_{\rm tb}}\times(+\sqrt{2gh}) = \frac{3m_{\rm bb}-m_{\rm tb}}{m_{\rm bb}+m_{\rm tb}}\times\sqrt{2gh}. \end{aligned} \tag{27}$$

The tennis ball is 10 times lighter then the basketball — $m_{\rm tb} = 57$ g while $m_{\rm bb} = 570$ g $= 10.0 \times m_{\rm tb}$ — hence

$$\begin{aligned} \frac{m_{\rm bb}-3m_{\rm tb}}{m_{\rm bb}+m_{\rm tb}} &= \frac{570\text{ g} - 3\times 57\text{ g}}{570\text{ g} + 57\text{ g}} = \frac{10.0-3.0}{10.0+1.0} \approx +0.64,\\ \frac{3m_{\rm bb}-m_{\rm tb}}{m_{\rm bb}+m_{\rm tb}} &= \frac{3\times 570\text{ g} - 57\text{ g}}{570\text{ g} + 57\text{ g}} = \frac{30.0-1.0}{10.0+1.0} \approx +2.64. \end{aligned} \tag{28}$$

Plugging these ratios into eqs. (27), we obtain

$$\begin{aligned} v'_{\rm bb} &\approx +0.64\times\sqrt{2gh} \approx +0.64\times 4.85\text{ m/s} = +3.1\text{ m/s},\\ v'_{\rm bb} &\approx +0.64\times\sqrt{2gh} \approx +0.64\times 4.85\text{ m/s} = +12.8\text{ m/s}. \end{aligned} \tag{29}$$

Note that after the second collision, both balls are moving up, although the tennis ball moves much faster than the basketball.

(d) After the second collision, each ball moves up until the gravity makes it fall down. The maximal height reached by a ball can be obtained from the free-fall equation of motion, but it is easier to use the mechanical energy conservation:

$$E = \tfrac{1}{2}mv^2 + mgy = \text{const.} \tag{30}$$

The ball starts with $v_{\rm init} = v'$ (velocity just after collision, *cf.* eqs. (29)) and height just above the floor, $y_{\rm init} \approx 0$, hence $E = \frac{1}{2}mv'^2 + 0$. When the ball reaches its maximal height $y^{\rm max}$, it has zero velocity, hence $E = 0 + mgy^{\rm max}$. By energy conservation

$$0 + mgy^{\rm max} = E = \tfrac{1}{2}mv'^2 + 0, \tag{31}$$

hence

$$y^{\rm max} = \frac{v'^2}{2g}\,. \tag{32}$$

Given the velocities (29) after the second collision, the basketball reaches maximal height

$$y_{\rm bb}^{\rm max} \;=\; \frac{v_{\rm bb}'^2}{2g} \;=\; \frac{(3.1\ {\rm m/s})^2}{2\times 9.8\ {\rm m/s^2}} \;=\; 0.49\ {\rm m}, \tag{33}$$

while the tennis ball flies much higher, to the maximal height of

$$y_{\rm tb}^{\rm max} \;=\; \frac{v_{\rm tb}'^2}{2g} \;=\; \frac{(12.7\ {\rm m/s})^2}{2\times 9.8\ {\rm m/s^2}} \;=\; 8.2\ {\rm m!} \tag{34}$$

In terms of the original drop height h, the basketball rebounds to

$$y_{\rm bb}^{\rm max} \;=\; \frac{\left(v_{\rm bb}' = 0.64\times\sqrt{2gh}\right)^2}{2g} \;=\; 0.64^2\times\frac{2gh}{2g} \;=\; 0.40\times h, \tag{35}$$

while the tennis ball bounces to

$$y_{\rm tb}^{\rm max} \;=\; \frac{\left(v_{\rm tb}' = 2.64\times\sqrt{2gh}\right)^2}{2g} \;=\; 2.64^2\times\frac{2gh}{2g} \;=\; 7.0\times h. \tag{36}$$

PS: In reality, the two collisions — of the basketball with the floor, and of the tennis ball with the basketball — are only approximately elastic, but some kinetic energy is lost each time. Consequently, the tennis ball does not quite jump to 7 times the initial drop height h. But as we saw in class, it does jump significantly higher than the height from which it was dropped, about 3 times higher, $y_{\rm tb}^{\rm max}\sim 3h$. Had I used a 57 g super-ball instead of the tennis ball and a 570 g super-ball instead of the beat-up basketball, the smaller super-ball on top would have jumped to almost $7h$ as in eq. (36). Or rather, it would have tried to jump that high, but would hit the ceiling before getting there.

Non-textbook problem #**IV**:

(a) The thrust of a rocket engine is a product of the exhaust speed (relative to the rocket) of the jet it produces and the mass flow rate of the jet,

$$F \;=\; u\times\frac{\Delta m}{\Delta t}\,. \tag{37}$$

In real rockets (unlike the toy I used in class), the jet is made of the burn product of the rocket fuel, so the mass flow rate of the jet is simply the rate at which the rocket burns its fuel and the oxidizer. For example, each main engine of a space shuttle burns 55 kg of liquid hydrogen and 440 kg of liquid oxygen each second, so its mass flow rate is

$$\left(\frac{\Delta m}{\Delta t}\right)_{\text{main engine}} = \frac{55 \text{ kg} + 440 \text{ kg}}{1 \text{ s}} = 495 \text{ kg/s}. \quad (38)$$

The exhaust speed of the main engine is 4440 m/s — the highest you can get from a chemical rocket without spewing very toxic HF into the atmosphere — so the thrust of each main engine is

$$F_{\text{main engine}} = u_{\text{main engine}} \times \left(\frac{\Delta m}{\Delta t}\right)_{\text{main engine}} = 4440 \text{ m/s} \times 495 \text{ kg/s} = 2.20 \cdot 10^6 \text{ N} \quad (39)$$

(about 250 tons of force).

The booster rockets have a smaller exhaust speed — about 2630 m/s — but they burn a lot more fuel every second, about 5200 kg/s, including both the fuel and the oxidizer. Hence, the thrust force of each booster is

$$F_{\text{booster}} = u_{\text{booster}} \times \left(\frac{\Delta m}{\Delta t}\right)_{\text{booster}} = 2630 \text{ m/s} \times 5200 \text{ kg/s} = 13.7 \cdot 10^6 \text{ N} \quad (40)$$

(about 1540 tons of force).

(b) At launch, the space shuttle assembly is lifted by five rocket engines, 3 main engines of the shuttle itself, and 2 solid rocket boosters. If all of them could operate at maximal power at launch, they would generate a combined lifting force

$$F_L = 3 \times F_{\text{main engine}} + 2 \times F_{\text{booster}} = 3 \times 2.20 \text{ MN} + 2 \times 13.7 \text{ MN} \approx 34.0 \text{ MN}. \quad (41)$$

However, once the shuttle lifts the ground, it is pulled down by its rather large weight

$$mg = 2.03 \cdot 10^6 \text{ kg} \times 9.8 \text{ N/kg} = 19.9 \text{ MN}, \quad (42)$$

so the net force lifting the shuttle is only

$$F_y^{\text{net}} = F_L - mg = 34.0 \text{ MN} - 19.9 \text{ MN} = 14.1 \text{ MN}. \quad (43)$$

Consequently, its starting acceleration is

$$a_y = \frac{F_y^{\text{net}}}{m} = \frac{14.1 \cdot 10^6 \text{ N}}{2.03 \cdot 10^6 \text{ kg}} = 6.95 \text{ m/s}^2. \quad (44)$$

PS: In reality, the rockets do not operate at full power at launch time because the atmospheric pressure reduces the exhaust speeds of the jets. At low altitudes, the main engine thrust is limited to 1.8 MN while the booster's thrust is only 12.5 MN, so the shuttle's lifts-off acceleration is only 4.3 m/s^2.

As the shuttle rises and the atmospheric pressure drops, the engine thrust increases to its vacuum values. At the same time, some of the fuel gets burned, so the shuttle's gross mass becomes smaller. Both factors increase the shuttle's acceleration to about 26 m/s^2 two minutes after the launch. At that point, the booster rockets run out of fuel and are dropped from the shuttle. The booster-less shuttle has less mass but also much less thrust, so its acceleration decreases to 8.4 m/s^2. But as the remaining fuel is burned up, the acceleration increases again, until about a minute before reaching the orbit, the acceleration reaches $3g = 29.4$ m/s. After that, the engines are throttled down a bit to keep the acceleration from getting too high and subjecting the astronauts to excessive g-forces.

Textbook problem **E8** at the end of chapter **8**:

(a) Since the 50 N force acts perpendicularly to the wrench's handle, is lever arm is the handle's length 24 cm. Consequently, the torque of this force is

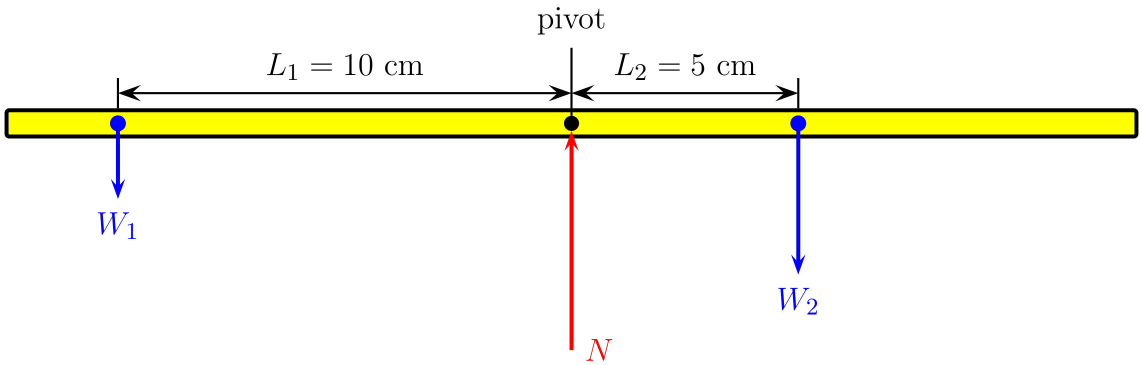
$$\tau \;=\; F \times L \;=\; 50\ \mathrm{N} \times 0.24\ \mathrm{m} \;=\; 12\ \mathrm{N} \times \mathrm{m}. \tag{45}$$

(b) The force applied in the middle of the handle rather than at its end has a smaller lever arm. Specifically, the force applied 12 cm from the nut and acting $\perp$ to the handle has lever arm of 12 cm. If the force has the same 50 N strength as before, its torque would be

$$\tau \;=\; F \times L \;=\; 50\ \mathrm{N} \times 0.12\ \mathrm{m} \;=\; 6\ \mathrm{N} \times \mathrm{m}. \tag{46}$$

PS: The metric unit of a torque is a Newton-meter — torque of a 1 Newton force of with a 1 meter lever arm. Superficially, this is similar to 1 Joule — the work of a 1 Newton force acting over a 1 meter distance. But in mechanical work, the distance is in the same direction as the force, while in torque, the lever arm is perpendicular to the torque. To maintain this distinction, the unit of torque is always called the Newton-meter rather than the Joule.

Textbook problem **E10** at the end of chapter **8**:



There are 3 forces acting on the balance — the two weights $W_1 = m_1 g$ and $W_2 = m_2 g$ placed on each side of the balance, and the normal force N at the pivot. The normal force N prevents the whole balance from falling down. But because N acts precisely at the pivot of the balance, it has zero lever arm and hence does not have any torque,

$$\tau(N) \;=\; 0. \tag{47}$$

But the other two forces W_1 and W_2 do have torques, and these torques must cancel each other, or else the balance will start tilting right or left.

The W_1 force is vertical, so its lever arm is the horizontal distance $L_1 = 10$ cm from the first weight to the pivot. The torque of this force is

$$\tau(W_1) \;=\; -W_1 \times L_1\,, \tag{48}$$

where the '−' sign indicates the clockwise direction of this torque.

Likewise, the W_2 force is vertical, so its lever arm is the horizontal distance $L_2 = 5$ cm from the second weight to the pivot. The torque of this force is

$$\tau(W_2) \;=\; +W_2 \times L_2\,, \tag{49}$$

where the '+' sign indicates the counterclockwise direction of this torque.

Altogether, the net torque on the balance is

$$\tau_{\text{net}} \;=\; \tau(N) \;+\; \tau(W_1) \;+\; \tau(W_2) \;=\; 0 \;-\; W_1 \times L_1 \;+\; W_2 \times L_2 \,. \tag{50}$$

To keep the balance horizontal this net torque must cancel, which calls for

$$W_1 \times L_1 \;=\; W_2 \times L_2 \,. \tag{51}$$

In this problem, we are given the two distances $L_1 = 10$ cm and $L_2 = 5$ cm and the first weight $W_1 = 8$ N. To find the second weight W_2 that would keep the balance horizontal, we solve eq. (51) for the W_2:

$$W_2 \;=\; \frac{W_1 \times L_1}{L_2} \;=\; \frac{8\ \text{N} \times 10\ \text{cm}}{5\ \text{cm}} \;=\; 16\ \text{N}. \tag{52}$$

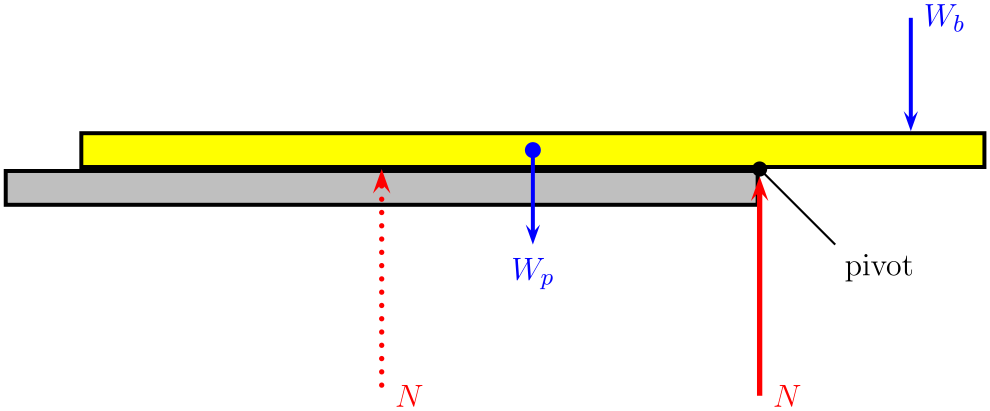
PS: Actually, there is a fourth force acting on the balance, namely its own weight mg. The lever arm of this weight is the horizontal distance from the pivot to the balance's *center of gravity*. In a bad balance, the center of gravity is to the left or to the right of the pivot, so the balance's own weight cause a torque. Such a bad balance would tilt left or right without any external weights attached to it.

In a good balance, the center of gravity should be right at the pivot — or vertically below the pivot — so that the balance's own weight has zero lever arm and does not make a torque. Consequently, the net torque on a good balance is precisely as in eq. (50).

For the purposes of this problem, I assume that the balance in question is good.

Textbook problem **SP2** at the end of chapter **8**:

There are 3 forces acting on the plank: its own weight W_p, the weight W_b of the boy that stands on it, and the normal force N from the dock:



(53)

The plank might pivot around the right end of the dock, so we should count all torques relative to this edge.

(a) The plank's own weight W_p is distributed all over the plank, but for the purposes of calculating the torque of that weight, we may assume it's concentrated at the plank's *center of gravity.* The picture in the textbook (diagram SP2 on page 167) shows the plank to be $3 + 1 = 4$ meters long. Assuming the plank is symmetric, its center of gravity is in its middle, 2 meters from each end. Relative to the dock's edge, the plank's center of gravity is at $x = +1 - 2 = -1$ m, that is, 1 meter to the left of the edge. Consequently, plank's weight W_p has lever arm $L_p = 1$ m relative to the pivot at the dock's edge, and the torque is *counterclockwise*

$$\tau(W_p) \;=\; +W_p \times L_p \;=\; +80\text{ N} \times 1\text{ m} \;=\; +80\text{ N} \times \text{m}. \tag{54}$$

(b) The lever arm of the boy's weight W_b is the boy's horizontal distance from the dock's edge. Counting the boy's position x from the dock's edge and assuming $x > 0$ — *i.e.*, the boy is to the right of the edge — his weight has lever arm $L_b = x$ and creates a *clockwise* torque

$$\tau(W_b) \;=\; -W_b \times x. \tag{55}$$

As to the normal force N from the dock, the place where it acts depends on the plank's position. If the plank lays flat on the dock, the normal force is distribute all over the surface of contact, and its torque has the same value as if N was acting at some *center of pressure* in the middle of this contact area; on the diagram (53), this is shown by the dotted red arrow. We don't know the exact location of the center of pressure, but we do know it lies to the left of the dock's edge, so the normal force N has a clockwise torque,

$$\tau(N) \; < \; 0. \tag{56}$$

On the other hand, if the plank is tilted to the right even a tiny bit, the contact between the plank and the dock is limited to the dock's right edge, so the whole normal force N acts at that point; on the diagram (53), this is shown by the solid red arrow. In this case, the normal force has zero lever arm relative to the edge, so its torque is zero,

$$\tau(N) \; = \; 0. \tag{57}$$

Altogether, all we can say about the torque of the normal force is that is can be zero or clockwise, but never counterclockwise,

$$\tau(N) \; \leq \; 0. \tag{58}$$

Now consider the net torque of all 3 forces,

$$\tau_{\text{net}} \; = \; \tau(W_p) \; + \; \tau(W_b) \; + \; \tau(N). \tag{59}$$

To keep the plank from tilting, this net torque must vanish, which requires the normal force to have torque

$$\tau(N) \; = \; -\tau(W_p) \; - \; \tau(W_b) \; = \; -80 \text{ N} \times \text{m} \; + \; W_b \times x. \tag{60}$$

But since this torque can be zero or clockwise but never counterclockwise, $\tau(N) \leq 0$, this formula gives us an inequality for the other two torques, namely

$$\tau(W_p) \ + \ \tau(W_b) \ = \ -80 \text{ N} \times \text{m} \ + \ W_b \times x \ = \ \tau(N) \ \leq \ 0. \tag{61}$$

In terms of the boy's position x, this inequality becomes

$$W_b \times x \ \leq \ -\tau(W_p) \ = \ +80 \text{ N} \times \text{m}, \tag{62}$$

hence

$$x \ \leq \ \frac{-\tau(W_p)}{W_b} \ = \ \frac{80 \text{ N} \times \text{m}}{150 \text{ N}} \ = \ 0.53 \text{ m}. \tag{63}$$

That is, the plank remains stable as long as the boy does not venture more than 53 cm beyond the dock's edge. If he goes further than that, the torque of his weight would be too large to be balanced by the torque of the plank's own weigh, so the plank will twist to the right — and the boy would end up in the water.

(c) Twisting of the plank takes time, and the smaller the net torque that makes it twist, the longer it takes. So the boy should walk on the plank very slowly, one very small step at a time. As soon as he feels the plank twisting under his feet, he should immediately take a big step back to safety, before the plank twists too far to drop him in the water.