Textbook problem **E18** at the end of chapter **8**:

In the absence of external torques, the student's *angular momentum*

$$L = I \times \omega \tag{1}$$

is conserved, *i.e.* keeps the same value regardless of what happens to the moment of inertia. When the student moves his arms close to his body, his moment of inertia becomes smaller, but his angular velocity increases so that the angular momentum stays the same,

$$I_2 \times \omega_2 = L = I_1 \omega_1 \tag{2}$$

and therefore

$$\omega_2 = \omega_1 \times \frac{I_1}{I_2}. \tag{3}$$

In this formula, ω_1 or ω_2 can be proper angular velocities (in radians per second) or rotation rates in any other units (such as RPS or RPM), as long as the same unit is used for both ω_1 and ω_2. Thus, for the student in question, the old rotation rate was $\omega_1 = 20$ RPM while the new rotation rate is

$$\omega_2 = 20 \text{ RPM} \times \frac{I_1 = 4.5 \text{ kg} \cdot \text{m}^2}{I_2 = 1.5 \text{ kg} \cdot \text{m}^2} = 60 \text{ RPM}. \tag{4}$$

Textbook problem **SP3** at the end of chapter **8**:

(a) The horizontal size of a child is rather small compared to his/her distance r from the axis of rotation, so to a good approximation his/her moment of inertia is simply

$$I_{\text{child}} \approx m \times r^2. \tag{5}$$

The net moment of inertia of several children is the sum their individual moments of inertia,

$$I_{\text{children}} = \sum_i^{\text{children}} I(i^{\text{th}} \text{ child}) \approx \sum_i^{\text{children}} m_i \times r_i^2. \tag{6}$$

The problem gives us the average distance of the children from the axis, but since it does not specify the technical meaning of *average* (median, or mean, or root-mean-square, or weighted average, or whatever), I am assuming that the individual distances are not much different from the average. Consequently,

$$\sum_{i}^{\text{children}} m_i \times r_i^2 \;\approx\; \sum_{i}^{\text{children}} m_i \times r_{\text{avg}}^2 \;=\; r_{\text{avg}}^2 \times \sum_{i}^{\text{children}} m_i \;=\; r_{\text{avg}}^2 \times M_{\text{all children}} \tag{7}$$

and hence

$$I_{\text{children}} \;\approx\; r_{\text{avg}}^2 \times M_{\text{all children}} \;=\; (2.0\text{ m})^2 \times 240\text{ kg} \;=\; 960\text{ kg}\cdot\text{m}^2. \tag{8}$$

The whole rotating system comprises the children and the merry-go-around itself, so its net moment of inertia is

$$I_{\text{net}} \;=\; I_{\text{mga}} + I_{\text{children}} \;=\; 1500\text{ kg}\cdot\text{m}^2 + 960\text{ kg}\cdot\text{m}^2 \;=\; 2460\text{ kg}\cdot\text{m}^2 \;\approx\; 2500\text{ kg}\cdot\text{m}^2. \tag{9}$$

(b) When the children move towards the axis of rotation, their moment of inertia becomes smaller. The new moment of inertia of the children is

$$I'_{\text{children}} \;=\; \sum_{i}^{\text{children}} m_i \times r_i'^2 \;\approx\; r_{\text{avg}}'^2 \times M_{\text{all children}} \;=\; (0.5\text{ m})^2 \times 240\text{ kg} \;=\; 60\text{ kg}\cdot\text{m}. \tag{10}$$

And the whole system (the children and the merry-go-around itself) has new moment of inertia

$$I'_{\text{net}} \;=\; I_{\text{mga}} + I'_{\text{children}} \;=\; 1500\text{ kg}\cdot\text{m}^2 + 60\text{ kg}\cdot\text{m}^2 \;=\; 1560\text{ kg}\cdot\text{m}^2 \;\approx\; 1600\text{ kg}\cdot\text{m}^2. \tag{11}$$

(c) In the absence of frictional torques, the net angular momentum of the system

$$L = I_{\text{net}} \times \omega \tag{12}$$

is conserved,

$$L' = L \implies I'_{\text{net}} \times \omega' = L' = L = I_{\text{net}} \times \omega. \tag{13}$$

As the net moment of inertia becomes smaller, the angular velocity must increase to keep the angular momentum constant, thus

$$\omega' = \omega \times \frac{I_{\text{net}}}{I'_{\text{net}}}. \tag{14}$$

For the system in question, the new angular velocity is

$$\omega' = \omega \times \frac{I_{\text{net}}}{I'_{\text{net}}} = 1.2\ \text{rad/s} \times \frac{2460\ \text{kg}\cdot\text{m}^2}{1560\ \text{kg}\cdot\text{m}^2} \approx 1.9\ \text{rad/s}. \tag{15}$$

(d) While the children walked closer to the axis, the angular velocity of the merry-go-around has increased from 1.2 rad/s (11 RPM) to 1.9 rad/s (18 RPM). In the process, the merry-go-around had angular acceleration

$$\alpha = \frac{\Delta\omega}{\Delta t}. \tag{16}$$

Although we don't know the time Δt the acceleration took and hence cannot calculate the numerical value of α, we know it was positive because the angular velocity has increased, $\Delta\omega > 0 \implies \alpha > 0$. And since the merry-go-around itself is rigid, this angular acceleration must have been caused by a torque

$$\tau[\text{on the mga}] = I_{\text{mga}} \times \alpha > 0. \tag{17}$$

There were no *external* torques on the whole system (merry-go-around plus children), so this torque on the merry-go-around itself must have come from the moving children. But how?

To answer this question, note that a child on a merry-go-around moves in a circle with speed

$$v = \omega \times r. \tag{18}$$

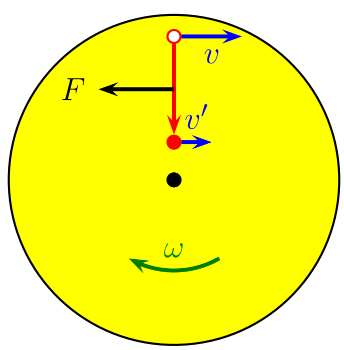
When the child moves closer to the axis, this speed decreases: although ω becomes larger, the radius r becomes smaller by a bigger factor. Indeed, a child at average distance from the axis had linear speed

$$v = 1.2 \text{ rad/s} \times 2.0 \text{ m} = 2.4 \text{ m/s} \tag{19}$$

before moving towards the axis, while after getting to the new average distance his/her speed has decreased to

$$v' = \omega' \times r' = 1.9 \text{ rad/s} \times 0.5 \text{ m} = 0.95 \text{ m/s}. \tag{20}$$

This, while a child is moving closer to the axis, his motion in the circular direction is *decelerating*, which requires a decelerating force $F = ma$:



The direction of this force is opposite to the circular motion, so it has a *negative torque on the child*. By the third law of Newton, there is an equal and opposite force — F in the positive circular direction — on the merry-go-around itself. This force creates a positive torque on the merry-go-around. *It is this torque — plus similar torques from the other children — that accelerates the merry-go-around's rotation when the children move towards the axis.*

Textbook problem **SP4** at the end of chapter **8**:

First, let's find out which components of the net angular momentum vector $\vec{\mathbf{L}}^{\rm net}$ of the student+stool+wheel system are conserved and which are not.

The student sits on a stool that freely rotates around a fixed vertical axis z. This means that there are *no external torques around this axis*, but there may be external torques around the horizontal axes x and y. In terms of the (τ_x,τ_y,τ_z) components of the eternal torque vector $\vec{\tau}^{\rm ext}$,

$$\tau_z^{\rm ext} = 0 \quad \text{but} \quad \tau_x^{\rm ext} \neq 0 \quad \text{and} \quad \tau_y^{\rm ext} \neq 0. \tag{21}$$

Each component of the torque vector changes the appropriate component of the net angular momentum of the student+stool+wheel system:

$$\Delta\vec{\mathbf{L}}^{\rm net} = \vec{\tau}^{\rm ext}\,\Delta t \quad\Longrightarrow\quad \begin{cases} \Delta L_x^{\rm net} = \tau_x^{\rm ext}\times\Delta t, \\ \Delta L_y^{\rm net} = \tau_y^{\rm ext}\times\Delta t, \\ \Delta L_z^{\rm net} = \tau_z^{\rm ext}\times\Delta t. \end{cases} \tag{22}$$

Hence, in light of eq (21) for the external torques, the z component of the net angular momentum is conserved,

$$L_z^{\rm net} = \text{const}, \tag{23}$$

while the other two components $L_x^{\rm net}$ and $L_y^{\rm net}$ may change with time.

In fact, the horizontal components $L_x^{\rm net}$ and $L_y^{\rm net}$ of the net angular momentum have to change when the student turns the wheel in all three dimensions. Since the student himself and the stool rotate only around the vertical axis, their angular momenta are vertical, thus

$$\begin{aligned} L_z^{\rm net} &= L_z^{\rm wheel} + L_z^{\rm student} + L_z^{\rm stool} \\ \text{but } L_x^{\rm net} &= L_x^{\rm wheel} \\ \text{and } L_y^{\rm net} &= L_y^{\rm wheel}. \end{aligned} \tag{24}$$

The wheel's own angular momentum $\vec{\mathbf{L}}^{\rm wheel}$ is parallel to the wheel's axis, so when the student turns this axis around, the horizontal components $L_x^{\rm wheel}$ and $L_y^{\rm wheel}$ are changing, and that changes the horizontal components $L_x^{\rm net}$ and $L_y^{\rm net}$ of the net angular momentum.

On the other hand, any changes in the vertical component L_z^{wheel} of the wheel's angular momentum are accompanied by the equal and opposite changes of the vertical angular momentum of the student+stool so that the net vertical angular momentum L_z^{net} is conserved.

(a) Initially, the student and the stool are at rest (not rotating) so their angular momentum is zero,

$$\vec{\mathbf{L}}^{\text{student+stool}} = \vec{\mathbf{0}} \quad \Longrightarrow \quad \vec{\mathbf{L}}^{\text{net}} = \vec{\mathbf{L}}^{\text{wheel}}. \tag{25}$$

The wheel spins on its axis with angular velocity $\omega = 5$ rev/s $= 31.4$ rad/s ≈ 30 rad/s so it has angular momentum

$$L^{\text{wheel}} = \omega \times I^{\text{wheel}} = 31.4\ \text{rad/s} \times 0.060\ \text{kg} \cdot \text{m}^2 = 1.885\ \text{N} \times \times \text{m} \approx 1.9\ \text{N} \times \times \text{m}. \tag{26}$$

The wheel's axis is vertical, so its angular momentum is also vertical. The diagram SP4 (on textbook page 167) shows this angular momentum pointing up, so

$$L_z^{\text{net}} = L_z^{\text{wheel}} = +\omega \times I^{\text{wheel}} \approx +1.9\ N \times s \times m, \tag{27}$$

and the wheel must be spinning counterclockwise (when viewed from above), *i.e.*, the side nearest to the student moves to his right while the distant side moves to his left.

(b) When the student flips the wheel's axis, the net vertical angular momentum L_z^{net} remains unchanged, but now it comes from several sources because everything is rotating: the wheel, the student, and the stool on which he sits. Moreover, the student's rotation makes the wheel's center go around the stool's axis while the wheel also spins around its own axis. Altogether,

$$L_z^{\text{net}} = L_z^{\text{student}} + L_z^{\text{stool}} + L_z^{\text{wheel's motion}} + L_z^{\text{wheel's spin}}. \tag{28}$$

Note the stool, the student, and the wheel in his hands rotate around the stools axis with the same angular velocity Ω (which we do not know yet but would like to calculate), so the net angular momentum of this rotation (not including the wheel's spin) is

$$L_z^{\rm SSW} \equiv L_z^{\rm student} + L_z^{\rm stool} + L_z^{\rm wheel's\ motion} = \Omega \times I^{\rm SSW} \tag{29}$$

where $I^{\rm SSW} = 6.0\ {\rm kg}\cdot{\rm m}^2$ is the moment of inertia of the whole system relative to the stools' axis.

As to the wheel's spin, its angular velocity ω has not changed so the angular momentum due to spin has the same *magnitude* as in part (a),

$$\left|\vec{\mathbf{L}}^{\rm wheel's\ spin}\right| = \omega \times I^{\rm wheel} \approx 1.9\ {\rm N} \times {\rm s} \times {\rm m}, \tag{30}$$

but the direction of this angular momentum is reversed: it points down rather than up, thus

$$L_z^{\rm wheel's\ spin} = -\omega \times I^{\rm wheel} \approx -1.9\ {\rm N} \times {\rm s} \times {\rm m}. \tag{31}$$

Indeed, the wheel is now spinning in the opposite direction relative to the student and to everybody else: clockwise (when viewed from above), *i.e.* the side nearest to the student moves to his left while the distant side moves to his right.

Altogether, the net vertical angular momentum of the whole system is

$$L_z^{\rm net} = L_z^{\rm SSW} + L_z^{\rm wheel's\ spin} = \Omega \times I^{\rm SSW} - \omega \times I^{\rm wheel}. \tag{32}$$

But since the z component of the net angular momentum is conserved, it must have the same value as in part (a), *i.e.* before the student has flipped the wheel's axis,

$$L_z^{\rm net} = +\omega \times I^{\rm wheel}. \tag{33}$$

Comparing these two equations for the $L_z^{\rm net}$, we see that we must have

$$\Omega \times I^{\rm SSW} - \omega \times I^{\rm wheel} = L_z^{\rm net} = +\omega \times I^{\rm wheel} \tag{34}$$

and therefore

$$\Omega \times I^{\rm SSW} = 2 \times \omega \times I^{\rm wheel}. \tag{35}$$

Solving this equation for the Ω — the angular velocity of the the whole system rotating on the stool's axis — we arrive at

$$\Omega \;=\; \omega \times 2 \times \frac{I^{\text{wheel}}}{I^{\text{SSW}}}\,. \tag{36}$$

Numerically,

$$\Omega \;=\; 5\ \text{rev/s} \times 2 \times \frac{0.060\ \text{kg}\cdot\text{m}^2}{6.0\ \text{kg}\cdot\text{m}^2} \;=\; 0.1\ \text{rev/s} \;=\; 6\text{rev/min}. \tag{37}$$

PS:

The original version of this problem in the textbook specified the wheel's moment of inertia as $I^{\text{wheel}} = 2\ \text{kg}\cdot\text{m}^2$, which is ridiculously large for a bicycle wheel. Plugging this number into eq. (36), we would get a much larger rotation rate of the student holding the flipped wheel, $\Omega = 3.3$ rev/s = 200 RPM. In real life, a student rotating 200 times a minute would fall off the stool and probably suffer a serious injury.

To avoid this nonsense, I told you to use a much smaller $I^{\text{wheel}} = 0.060\ \text{kg}\times\text{m}^2$, which is in the right ballpark for a wire-spoke bicycle wheel. Consequently, the answer to part (b) — student's rotation rate $\Omega = 6$ RPM — is also quite realistic. Indeed, when I demonstrated flipping a spinning wheel in class, the student started rotating at $\Omega \sim$ a few revolutions per minute.

(c) Turning the axis of the spinning wheel changes the direction of its angular momentum. Any change in the angular momentum vector requires a torque according to

$$\Delta\vec{\mathbf{L}} \;=\; \vec{\tau}\,\Delta t, \tag{38}$$

so the student must apply a torque to the wheel's axis to make it change direction. By the rotational analog of the Newton's third law, the wheel applies an equal and opposite torque to the student. It is this torque — or rather its z component — that gives the student and the stool their angular acceleration.

Non-textbook problem #**I**:

(a) The angular momentum vector of the spinning wheel is parallel to the wheel's axis. Along that axis, the direction of the angular momentum is given by the right screw rule: if from your point of view the wheel turns counterclockwise, the $\vec{L}$ vector points towards you, but if the wheel turns clockwise, $\vec{L}$ points away from you.

For the wheel shown on the picture, the axis is horizontal; its near end points towards the camera and slightly to the right while the far end points away and slightly to the left. Since the wheel turns clockwise, the right screw rule tells us that the angular momentum vector points in the direction of the far end of the axis: Thus, $\vec{L}$ points slightly to the left of the horizontal direction away from the camera.

(b) The force acting on the wheel are its weight mg and the tension T of the string tied to the near end of the axis. Both forces are vertical, but they do not act along the same vertical line, so they create a net torque.

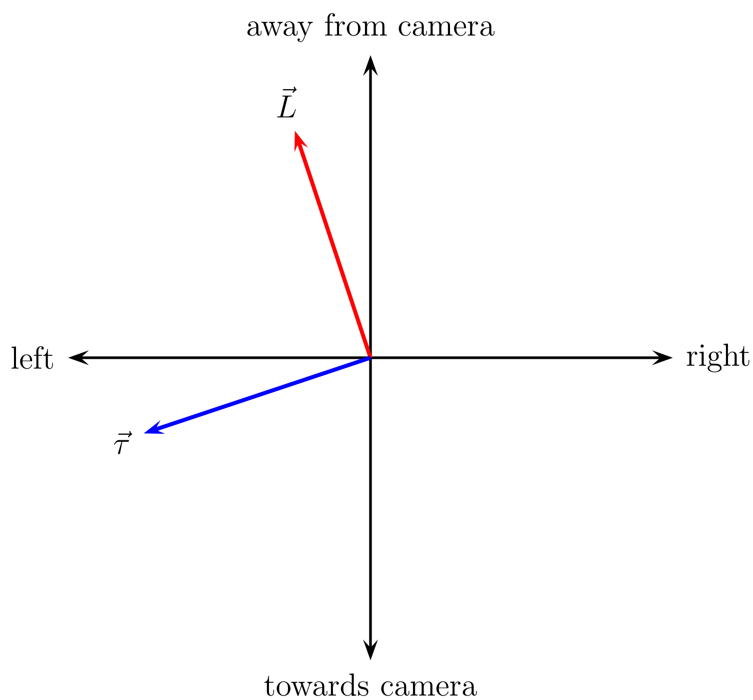
Since the net force on the wheel is zero, we may calculate the net torque relative to any pivot point we like, so let's use the point where the string is tied to the axis as a pivot. Relative to this pivot, the string tension no torque, so the net torque comes from just from the wheels weight force $\vec{F} = m\vec{g}$. The direction of this force is vertically down, while the point at which it acts — the wheel's center of gravity, located at the wheel's geometric center — has radius vector $\vec{R}$ (relative to the pivot) which points in the direction of the wheel's axis.

The torque vector $\vec{\tau} = \vec{R} \times \vec{F}$ is perpendicular to both $\vec{R}$ and $\vec{F}$ vector. Vectors $\perp \vec{R}$ — and hence $\perp$ to the wheel's axis — lie in the plane of wheel itself, while vectors $\perp$to the vertical force $m\vec{g}$ are horizontal. Combining the two conditions, we find that the torque vector $\vec{\tau}$ points horizontally along the wheel. For the wheel shown on the picture, this means either horizontally to the right and slightly away from the camera, or horizontally to the left and slightly towards the camera.

To chose between these two opposite directions, we use the right screw rule. A person looking at the wheel from the left side would see the wheel's weight trying to turn the wheel counterclockwise. A right screw turned in that direction would go out, towards the observer, so by the right screw rule, the torque vector $\vec{\tau}$ points towards that observer. Since that observer looks from the left side of the wheel, from the camera's point of view $\vec{\tau}$ points to the left.

The bottom line is, the torque vector $\vec{\tau}$ points horizontally to the left and slightly towards the camera.

(c) The angular momentum vectors $\vec{L}$ and the torque vector $\vec{\tau}$ are both horizontal, so let's draw them in the horizontal plane:



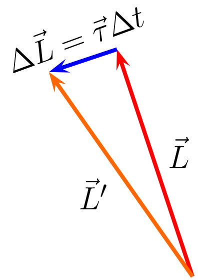
Note that the torque vector is $\perp$ to the angular momentum vector.

Over a short period of time, the angular momentum changes by

$$\Delta\vec{L} \;=\; \vec{\tau}\,\Delta t. \tag{39}$$

This is a vector equation, so the direction of $\Delta\vec{L}$ is the direction of the torque vector.

Diagrammatically



where $\vec{L}$ is the angular momentum vector at the time the picture of the wheel was taken while $\vec{L}'$ is the angular momentum vector after a shot time interval Δt. We see that the direction of the $\vec{L}'$ is to the left of the $\vec{L}$. Thus, *the torque makes the angular momentum vector turn left.*

But the direction of the wheel's angular momentum vector is the direction of the wheel's axis. If $\vec{L}$ turns left, this means that *the axis turns left.*

Non-textbook problem #**II**:

At the ocean's surface, the water pressure is equal to the atmospheric pressure,

$$P_{\text{surface}} = P_{\text{atm}} \approx 1 \text{ atm} = 760 \text{ Torr} = 101\,300 \text{ Pa}. \tag{40}$$

At the bottom of the Challenger Deep, the pressure increases by the weight of almost 7 miles of water above it,

$$P_{\text{bot}} - P_{\text{surface}} = \rho_{\text{seawater}} \times g \times d. \tag{41}$$

The density of the seawater depends on the depth: near the surface $\rho \approx 1027 \text{ kg/m}^3$, 3 miles below the surface (average depth of the Pacific ocean) the density increases to 1050 kg/m^3, while near the bottom of the Challenger Deep it reaches 1073 kg/m^3. For the purpose of this calculation we use the average density, about 1050 kg/m^3, about 5% denser than the fresh water, thus

$$\begin{aligned} P_{\text{bot}} - P_{\text{surface}} &= \rho_{\text{seawater}} \times g \times d_{\text{Deep}}^{\text{Challenger}} \\ &= 1050\ \text{kg/m}^3 \times 9.8\ \text{N/kg} \times 10923\ \text{m} \\ &= 112\,400\,000\ \text{Pa} = 1110\ \text{atm}, \end{aligned} \tag{42}$$

and hence *water pressure at the bottom of the Challenger Deep is about 1111 atmospheres* or 112.5 mega-Pascal.

Textbook question **Q5** at the end of chapter **9**:

In a round tire, the force on a rubber due to higher air pressure inside the tire than outside it is counteracted by the tension of the rubber. However, when the tire is pushed into the ground by the vehicle's weight, the small area at the bottom of the tire is flattened. Over this flattened area — called the *contact patch* because that's where the rubber contacts the road — the rubber's tension does not counteract the force of the air pressure; instead, the air pressure provides the normal force between the tire and the ground.

$$N = P^{\text{gauge}} \times A \tag{43}$$

In this formula

$$P^{\text{gauge}} = P^{\text{air inside}} - P^{\text{air outside}} \tag{44}$$

is the *gauge pressure* of the air inside the tire while A is the area of the contact patch.

The pressure to which you need to inflate your tires is

$$P^{\text{gauge}} = \frac{N}{A}. \tag{45}$$

A bicycle tire bears much smaller normal force N than a car tire. but it is also much thinner, so its contact patch has a much smaller area A. It turns out that the smaller area has a bigger effect than the smaller force, so the gauge pressure in a bicycler tire should be higher than in a car tire.

Indeed, a typical car tire is 7 to 9 inches wide. It is also rather thick, so the flattened contact patch at the bottom can be up to 7 inches long without damaging the tire, although this would make the tire wear out faster and increase the rolling resistance. Optimal inflation of the tire keeps the contact patch length down to 2–3 inches, so its area $A = \text{width} \times \text{length}$ is about 20 to 30 square inches; for the sake of definiteness, let's assume $A = 25\ \text{inch}^2$. The gross weight of a car can be anywhere between 2000 pounds (an empty sub-compact) to 5000 pounds (a full-sized car carrying 5 big passengers and a lot of cargo). For the sake of definiteness, let's assume $Mg = 3000$ pounds. Each tire carries 1/4 of this weight, thus $N = 750$ pounds, which calls for gauge pressure in the tire

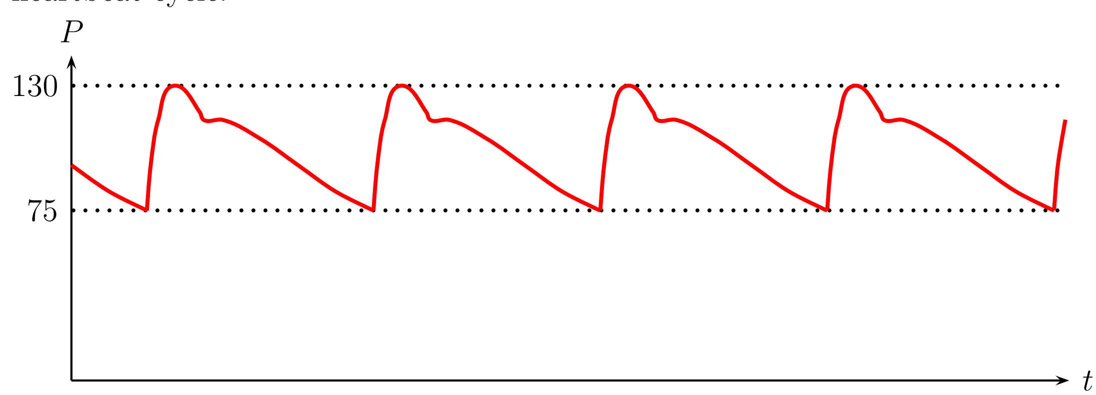
$$P_{\text{car}}^{\text{gauge}} = \frac{N \sim 750\ \text{lb}}{A \sim 25\ \text{inch}^2} \sim 30\ \text{lb/inch}^2 \equiv 30\ \text{PSI}. \tag{46}$$

A bicycle tire is much narrower, less than an inch wide. Also, it is much thinner so the flattened contact patch cannot be much longer than 2 inches. Consequently, the contact patch area A is about 1 square inch. (It could be a bit larger or a bit smaller, but for the sake of definiteness let's assume $A = 1\ \text{inch}^2$.) So if the gross weight of the rider and the bike is 160 pounds and each wheel carries half of that weight, $N = \frac{1}{2}mg = 80$ lb, you should inflate your bike's tires to gauge pressure

$$P_{\text{bike}}^{\text{gauge}} = \frac{N \sim 80\ \text{lb}}{A \sim 1\ \text{inch}^2} \sim 80\ \text{lb/inch}^2 \equiv 80\ \text{PSI}. \tag{47}$$

Textbook question **Q10** at the end of chapter **9**:

First of all, a blood pressure reading of 130/75 does *not* mean 130 divided by 75. Instead, the two numbers — 130 and 75 — give the range of the blood pressure as it changes during the heartbeat cycle:



In this example, 130 is *the high pressure*, reached when the heart pumps the blood out to the aorta and hence down the arteries, while 75 is *the low pressure* reached when the heart relaxes before the next beat. In medical literature, the high pressure is called the systolic pressure while the low pressure is called the diastolic pressure.

Second, the blood pressure is always measured in units of *millimeters of mercury*,

$$1\ \text{mm Hg} \;=\; 1\ \text{mm} \times \rho_{\text{mercury}} \times g \;\approx\; 133\ \text{Pa}. \tag{48}$$

Thus, BP = 130/75 means the high pressure is 130 millimeters of mercury while the low pressure is 75 millimeters of mercury.

Finally, the blood pressure the doctors measure and care about is a gauge pressure — the difference between the absolute pressure of the blood in the patient's arteries and the atmospheric pressure outside the patient's body. It's the gauge pressure that pushes on the arteries' walls, it's the gauge pressure that makes the blood circulate through the patient's body, and the patient's heart works to give the blood its gauge pressure — the atmospheric pressure comes for free.

Given the gauge pressure of the blood, you can easily compute its absolute pressure by adding the atmospheric pressure, for example $130 + 760 = 890$ millimeters of mercury. But this absolute pressure depends more on the altitude and the weather than on the patient's health, so the doctors don't care about it, they want to know the gauge pressure.

Textbook problem **E5** at the end of chapter **9**:
The pushing piston applies force $F_1 = 400$ N (about 90 pounds) over the area $A_1 = 0.001\ \text{m}^2$ (10 cm^2 or about 1.5 square inch), so its pressure on the hydraulic fluid (presumably some kind of oil) is

$$P \;=\; \frac{F_1}{A_1} \;=\; \frac{400\ \text{N}}{1 \cdot 10^{-3}\ \text{m}^2} \;=\; 400\,000\ \text{Pa} \;\approx\; 4\ \text{atm}. \tag{49}$$

To be precise, this is the *gauge pressure* of the hydraulic fluid, *i.e.* the difference between its absolute pressure and the air pressure outside the vessel.

By Pascal's Law, the load-bearing piston experiences the same gauge pressure. This piston has a much bigger area $A_2 = 0.2\ \mathrm{m}^2 = 2000\ \mathrm{cm}^2 \approx 300\ \mathrm{inch}^2$, so the force on this piston is much larger,

$$F_2 \;=\; P \times A_2 \;=\; 400\,000\ \mathrm{Pa} \times 0.2\ \mathrm{m}^2 \;=\; 80\,000\ \mathrm{N}, \tag{50}$$

about 9 tons.