

Non-textbook problem #I:

Suppose the total volume of alcohol in the thermometer is V_0 . When the temperature changes by ΔT , the alcohol volume changes by

$$\Delta V = V_0 \times \beta \times \Delta T \quad (1)$$

where $\beta = 1.12 \cdot 10^{-3} (\text{°C})^{-1}$ is the alcohol's thermal expansion coefficient. If the glass does not expand, the internal volume of the bulb stays fixed, so the additional alcohol volume goes into the cylinder where it raises the alcohol level by

$$\Delta h = \frac{\Delta V}{A} = \frac{V_0 \times \beta \times \Delta T}{A} \quad (2)$$

where $A = \pi r^2 = 0.25 \text{ mm}^2$ is the cylinder's internal cross-section. Consequently, the ratio of the alcohol level rise to the temperature rise is

$$\frac{\Delta h}{\Delta T} = \frac{V_0 \times \beta}{A}. \quad (3)$$

But we know this ratio for the thermometer in question,

$$\frac{\Delta h}{\Delta T} = 1 \text{ mm/°C}, \quad (4)$$

so we may use eq. (3) to find the net volume v_0 of alcohol in the thermometer:

$$V_0 = \frac{A}{\beta} \times \frac{\Delta h}{\Delta T} = \frac{0.25 \text{ mm}^2}{1.12 \cdot 10^{-3} (\text{°C})^{-1}} \times 1 \text{ mm/°C} = 223 \text{ mm}^3. \quad (5)$$

Most of this alcohol is inside the bulb since the cylinder has much smaller volume. Indeed, the alcohol level showed on the picture is only 16 mm above the bulb,[★] so the volume of alcohol inside the cylinder is only $h \times A = 16 \text{ mm} \times 0.25 \text{ mm}^2 = 4 \text{ mm}^3$. The remaining $223 - 4 = 219 \approx 220 \text{ mm}^3$ of alcohol is inside the bulb, so its internal volume must be 220 mm^3 .

[★] The thermometer shows -32°C (I presume the picture was taken in Minnesota winter) while the top of the bulb is at the -48°C level. The 16°C difference corresponds to 16 mm of height.

Non-textbook problem #II:

The density of Venus atmosphere is the mass to volume ratio of some amount of gas,

$$\rho = \frac{M(\text{some amount of gas})}{V(\text{the same amount of gas})} \quad (6)$$

We can use any amount of gas we like in this formula, as long as take both the mass and the volume of the same amount. So let's find the mass and the volume of 1 mol of Venus atmosphere.

The mass (in grams) of one mol of any substance is its molecular weight, which depends on the chemical composition and nothing else. The atmosphere of Venus consists mostly of carbon dioxide, so its molecular weight is $\mu(\text{Venus atmosphere}) \approx \mu(\text{CO}_2) = 44$. Therefore, the mass of one mol is

$$M(1 \text{ mol}) = 44 \text{ g}. \quad (7)$$

On the other hand, the volume of one mol of a gas depends on its temperature and pressure but does not depend on its chemical composition. Indeed, according to the Universal Gas Law,

$$P \times V = n \times R \times T \quad (8)$$

where V is the volume of the gas, P is its pressure, T is its absolute temperature, $R = 8.314 \text{ J/}^\circ\text{K/mol}$ is a universal gas constant, same for all gases, and n is the amount of gas in mols. Solving eq. (8) for the volume of the gas, we obtain

$$V = \frac{n \times R \times T}{P}. \quad (9)$$

In particular, under conditions prevailing on Venus just above the planet's surface, one mol of Venus atmosphere occupies volume

$$V(1 \text{ mol}) = \frac{(1 \text{ mol}) \times (8.314 \text{ J/K/mol}) \times (740 \text{ K})}{(9.2 \cdot 10^6 \text{ Pa})} = 670 \cdot 10^{-6} \text{ m}^3 = 670 \text{ cm}^3. \quad (10)$$

At this point, all we need to do is to combine eqs. (7) and (10): the density of the Venus

atmosphere (near the surface) is

$$\rho = \frac{M(1 \text{ mol})}{V(1 \text{ mol})} = \frac{44 \text{ g}}{670 \text{ cm}^3} = 0.066 \text{ g/cm}^3 = 66 \text{ kg/m}^3.$$

Non-textbook problem #III:

Initially, the wort is at $T_w^0 = 35^\circ\text{C}$ and the rocks are at $T_r^0 = 360^\circ\text{C}$. After the rocks and the wort reach thermal equilibrium, they all have the same temperature T^e — and that's what we need to calculate.

During the process (of reaching the equilibrium) the rocks cool down from T_r^0 to T^e , which releases the amount of heat given by

$$Q_r = C_r \times (T_r^0 - T^e) \quad (11)$$

where C_r is heat capacity of the rocks. In terms of the rock's mass M_r and specific heat c_r ,

$$C_r = M_r \times c_r = 20 \text{ kg} \times 0.19 \text{ cal/g/}^\circ\text{C} = 3.8 \text{ kcal/}^\circ\text{C}. \quad (12)$$

At the same time, the wort warms up from T_w^0 to T^e , which consumes the amount of heat given by

$$Q_w = C_w \times (T^e - T_w^0) \quad (13)$$

where C_w is heat capacity of the wort. In terms of the mass M_w and specific heat c_w of the wort,

$$C_w = M_w \times c_w = 20 \text{ kg} \times 1.04 \text{ cal/g/}^\circ\text{C} = 20.8 \text{ kcal/}^\circ\text{C}. \quad (14)$$

The heat warming up the wort comes from the cooling rocks, thus $Q_w = Q_r$ and therefore

$$C_w \times (T^e - T_w^0) = C_r \times (T_r^0 - T^e). \quad (15)$$

Solving this equation for the equilibrium temperature T^e , we find

$$T^e \times (C_w + C_r) = T_w^0 \times C_w + T_r^0 \times C_r \quad (16)$$

and therefore

$$\begin{aligned} T^e &= \frac{C_w}{C_w + C_r} \times T_w^0 + \frac{C_r}{C_w + C_r} \times T_r^0 \\ &= \frac{20.8 \text{ kcal}/^\circ\text{C}}{20.8 \text{ kcal}/^\circ\text{C} + 3.8 \text{ kcal}/^\circ\text{C}} \times 35^\circ\text{C} + \frac{3.8 \text{ kcal}/^\circ\text{C}}{20.8 \text{ kcal}/^\circ\text{C} + 3.8 \text{ kcal}/^\circ\text{C}} \times 360^\circ\text{C} \\ &= 0.8455 \times 35^\circ\text{C} + 0.1545 \times 360^\circ\text{C} \\ &= 85.2^\circ\text{C}. \end{aligned} \tag{17}$$

Non-textbook problem #IV:

(a) A 100 g bullet moving at speed 400 m/s has kinetic energy

$$K = \frac{1}{2}M \times v^2 = \frac{1}{2} \times 0.1 \text{ kg} \times (400 \text{ m/s})^2 = 8000 \text{ J}. \tag{18}$$

When the bullet hits the target and stops, all this energy is converted to heat. Some of that heat goes towards warming the ice from -20°C to the melting point 0°C , while the remaining heat melts the some of the ice. The amount of heat used up by warming the ice is

$$Q^{\text{warm ice}} = c_{\text{ice}} \times M \times \Delta T = 2.108 \text{ J}/^\circ\text{C/g} \times 100 \text{ g} \times (0^\circ\text{C} - (-20^\circ\text{C})) = 4216 \text{ J} \tag{19}$$

so the remaining heat that melts the ice is only

$$Q^{\text{melt ice}} = K - Q^{\text{warm ice}} = 8000 \text{ J} - 4216 \text{ J} = 3784 \text{ J}. \tag{20}$$

(Only the first figure in this number is reliable, but I keep extra figures at intermediate stages to avoid round-off errors.) Melting the ice takes latent heat in the amount $Q = m \times L_m$ where $L_m = 334 \text{ J/g}$, so the mass of ice melted by the remaining heat (20) is

$$M^{\text{melted}} = \frac{Q^{\text{melt ice}}}{L_m} = \frac{3784 \text{ J}}{334 \text{ J/g}} \approx 11 \text{ g}. \tag{21}$$

(b) To completely vaporize the bullet, we need to warm the ice to the melting point, then melt all the ice, then warm up the water from 0°C to 100°C , and then vaporize all the water. The minimal amount of heat needed to vaporize the whole bullet is therefore

$$\begin{aligned}
Q^{\min} &= Q^{\text{warm ice}} + Q^{\text{melt ice}} + Q^{\text{warm water}} + Q^{\text{vaporize water}} \\
&= M \times c_{\text{ice}} \times (0^\circ\text{C} - (-20^\circ\text{C})) + M \times L_m \\
&\quad + M \times c_{\text{water}} \times (100^\circ\text{C} - 0^\circ\text{C}) + M \times L_v \\
&= M \times \left[2.108 \text{ J}/^\circ\text{C}/\text{g} \times 20^\circ\text{C} + 334 \text{ J}/\text{g} + 4.187 \text{ J}/^\circ\text{C}/\text{g} \times 100^\circ\text{C} + 2257 \text{ J}/\text{g} \right] \\
&= M \times 3052 \text{ J}/\text{g} = M \times 3.052 \text{ MJ}/\text{kg}.
\end{aligned} \tag{22}$$

Note that this is the minimal amount; for $Q > Q^{\min}$, the whole bullet vaporizes, and then the excess heat goes to increase the vapor's temperature above the boiling point 100°C .

This means that if this heat comes from the kinetic energy of the bullet, we need at least 3.014 Megajoules of kinetic energy per kilogram of bullet's mass. But the ratio of kinetic energy to mass is

$$\frac{K}{M} = \frac{\frac{1}{2}Mv^2}{M} = \frac{v^2}{2}, \tag{23}$$

so minimal kinetic energy per unit of mass means minimum speed v_{\min} such that

$$\frac{v^2}{2} > \frac{v_{\min}^2}{2} = \frac{K^{\min}}{M} = \frac{Q^{\min}}{M} = \frac{3052 \text{ J}}{1 \text{ g} = 0.0001 \text{ kg}} = 3.052 \cdot 10^6 \text{ J}/\text{kg} = 3.052 \cdot 10^6 \text{ m}^2/\text{s}^2, \tag{24}$$

hence

$$v > v_{\min} = \sqrt{2 \times 3.052 \cdot 10^6 \text{ m}^2/\text{s}^2} = 2460 \text{ m}/\text{s} \approx 5300 \text{ mile}/\text{hour}. \tag{25}$$

Note that this minimal speed does not depend on the bullet's mass: Any amount of ice — from a snowflake to an icy asteroid — will be completely vaporized on impact if it hits a hard target at this speed or faster.

Non-textbook problem #V:

Before we start any calculation, please note that food calories are big Calories or kilocalories,

$$1 \text{ Cal} = 1 \text{ kcal} = 1000 \text{ cal} = 4187 \text{ J}. \quad (26)$$

So when we say that a spoonful of sugar has 20 calories, we actually mean 20 000 cal or 83 500 J of chemical energy. If this energy were converted to heat, it would suffice to bring a cup of cold water to a boiling point. In a human body, this energy could be used to stay warm in cold weather, or to power muscles doing mechanical work — or it could be stored as chemical energy of a different substance, namely fat.

When the chemical energy of food is used to power the muscles, about one quarter of that energy goes to mechanical work while the remaining three quarters turn into heat. Thus, *for each food calorie burned by the muscles they produce $\frac{1}{4} \times 4187 \text{ J} \approx 1050 \text{ J}$ while the remaining $\frac{3}{4} \text{ Cal} = 750 \text{ cal}$ turn into heat.*

Riding a bike at low speeds takes little power (only 30 W for riding at 10 mph on flat ground), but increasing the speed causes strong air drag force which takes much larger power to overcome: For an average rider on an average bike, riding at 20 mph over flat ground takes about 175 W of human power. Producing this power for an hour takes mechanical energy

$$W = P \times t = 175 \text{ W} \times 3600 \text{ s} = 630\,000 \text{ J}. \quad (27)$$

To produce this mechanical work, human muscles need four times as much food energy,

$$E = 4W = \frac{630\,000 \text{ J}}{\frac{1}{4} \times 4187 \text{ J/Cal} \approx 1050 \text{ J/Cal}} \approx 600 \text{ Cal}, \quad (28)$$

i.e., 600 food calories.

Besides mechanical work, burning these calories produces heat in the amount of

$$Q = \frac{3}{4} \times E = 450 \text{ Cal} = 450\,000 \text{ cal}. \quad (29)$$

The body gets rid of this heat by sweating. Evaporating one gram of sweat at body temperature takes 580 calories (the little calories, not the food calories), so the amount of sweat

the body needs to produce and vaporize to lose all the heat (29) is

$$m_{\text{sweat}} = \frac{Q}{L_v} = \frac{450\,000 \text{ cal}}{580 \text{ cal/g}} \approx 775 \text{ g.} \quad (30)$$

Non-textbook problem #VI (optional exercise):

(a) The work done by the expanding gas pushing on the piston is

$$W = P \times \Delta V \quad (31)$$

For a gas held at constant pressure, $P \times \Delta V = \Delta(PV)$, hence in light of the universal gas law

$$P \times V = n \times R \times T \quad (32)$$

the mechanical work (31) can be expressed as

$$W = P \times \Delta V = \Delta(PV) = \Delta(nRT) = n \times R \times \Delta T. \quad (33)$$

For the one mol of air in question, the work is

$$W = 1 \text{ mol} \times 8.314 \text{ J/K/mol} \times (500 \text{ K} - 300 \text{ K}) \approx 1660 \text{ J.} \quad (34)$$

(b) The internal energy of a diatomic gas — or a mixture of diatomic gases such as air — is

$$U = \frac{5}{2} \times n \times R \times T \quad (35)$$

where n is the amount of gas in mols, T is the absolute temperature, and R is the universal gas constant. Note that U does not depend on the volume of the gas, so when the gas temperature increases under any conditions — fixed volume, or fixed pressure, or whatever — the internal energy of a diatomic gas increases by

$$\Delta U = \frac{5}{2} \times n \times R \times \Delta T. \quad (36)$$

For the one mol of air in question,

$$\Delta U = \frac{5}{2} \times 1 \text{ mol} \times 8.314 \text{ J/K/mol} \times (500 \text{ K} - 300 \text{ K}) \approx 4160 \text{ J.} \quad (37)$$

(c) The First Law of Thermodynamics says that the net energy — counting all forms of energy and all the bodies involved — is always conserved. In particular, the net energy of a body (such as a mol of air) changes by the net inflow of energy, including both the heat flowing into the body and the mechanical work done on it. The mol of air in question does not move so its overall kinetic or potential energies do not change. Only the internal energy of the air is changing, hence

$$\Delta U = \Delta E^{\text{net}} = Q^{\text{in}} + W^{\text{in}} \quad (38)$$

where Q^{in} is the amount of heat transferred to the air and W^{in} is the mechanical work of external forces (*i.e.*, piston's pressure) on the air. In term of the mechanical work $W^{\text{out}} = P\Delta V$ done by the air on the piston,

$$W^{\text{in}} = -W^{\text{out}} = -P\Delta V, \quad (39)$$

thus

$$\Delta U = Q^{\text{in}} - P \times \Delta V. \quad (40)$$

In parts (a) and (b) of this problem we have calculated the mechanical work $P\Delta V$ and the change of internal energy, so we may solve eq. (40) for the amount of heat transferred to the air to raise its temperature:

$$Q^{\text{in}} = \Delta U + P \times \Delta V = 4160 \text{ J} + 1660 \text{ J} = 5820 \text{ J}. \quad (41)$$

(d) The heat needed to change the air's temperature is the sum of its internal energy change ΔU and the mechanical work $P \times \Delta V$ done by the expanding air. The ΔU does not depend on the pressure at which the expanding air was held because the internal energy of air depends only on its temperature and does not depend on the volume. As to the mechanical work, we saw in part (a) that for gas held at *any constant pressure*

$$W^{\text{out}} = P \times \Delta V = \Delta(PV) = \Delta(nRT) = n \times R \times \Delta T. \quad (42)$$

Thanks to the universal gas law, this work depends only on the amount of gas (in mols) and the overall change of its temperature but it does not depend on the specific pressure at which the gas was held, as long as that pressure was constant.

Consequently, the amount of heat needed to change the air's temperature does not depend on the the specific pressure at which the gas was held, as long as that pressure was constant.

(e-f) When the air is held at constant volume it does not do any mechanical work. Indeed,

$$V = \text{const} \implies \Delta V = 0 \implies W^{\text{out}} = P \times \Delta V = 0. \quad (43)$$

Consequently, all the heat transferred to the air goes towards increasing its internal energy,

$$\Delta U = Q^{\text{in}}. \quad (44)$$

Since the internal energy of air does not depend on its volume, ΔU does not depend on the volume at which the air is held but only on the change in temperature,

$$\Delta U = \frac{5}{2} \times n \times R \times \Delta T. \quad (45)$$

Hence, *the amount of heat needed to change the temperature of air held at fixed volume does not depend on the value of that volume, as long as it stays constant.* For the one mol of air in question, ΔU is the same as in part (b) while $W^{\text{out}} = 0$, thus

$$Q^{\text{in}} = \Delta U \approx 4160 \text{ J}. \quad (46)$$

(g) In previous parts we saw that the amount of heat needed to change air's temperature by a particular ΔT depends on whether the air is held at constant volume or constant pressure, but it does not depend on the specific values of the constant volume or constant pressure. Thus, the air —like all the gases— has two different molar heat capacities (heat capacities of one mol), namely C_p for the air held at constant pressure and C_v for the air held at constant

volume. Numerically,

$$C_p = \frac{Q^{\text{in}}[\text{in part (c)}]}{\Delta T} \approx \frac{5820 \text{ J/mol}}{200 \text{ K}} = 29.1 \text{ J/K/mol} \quad (47)$$

while

$$C_v = \frac{Q^{\text{in}}[\text{in part (e)}]}{\Delta T} = \frac{4160 \text{ J/mol}}{200 \text{ K}} = 20.8 \text{ J/K/mol}. \quad (48)$$

Analytically, for the gas held at constant volume

$$Q^{\text{in}} = \Delta U = \frac{5}{2} \times n \times R \times \Delta T, \quad (49)$$

hence

$$C_v = \frac{Q^{\text{in}}}{n \times \Delta T} = \frac{5}{2} \times R. \quad (50)$$

For the gas held at constant pressure,

$$W^{\text{out}} = P \times \Delta V = n \times R \times \Delta T, \quad (51)$$

hence

$$Q^{\text{in}} = \Delta U + P \times \Delta V = \frac{5}{2} \times n \times R \times \Delta T + n \times R \times \Delta T = \frac{7}{2} \times n \times R \times \Delta T \quad (52)$$

and therefore

$$C_p = \frac{Q^{\text{in}}}{n \times \Delta T} = \frac{7}{2} \times R. \quad (53)$$

PS: The other gases have different molar heat capacities C_v and C_p because of different different internal energies $U(T)$. For example, the monoatomic gases such as helium have $C_v = \frac{3}{2}R$ and $C_p = \frac{5}{2}R$. However, all gases have the same difference $C_p - C_v = R$.