1. First, a reading assignment: §16.5 of the *Peskin & Schroeder* textbook. Make sure you understand the calculation of the (infinite parts of) the counterterms $\delta_1$, $\delta_2$ and $\delta_3$ in detail.

2. Next, another reading assignment: §20.1 of the *Peskin & Schroeder* textbook, which introduces the *Higgs mechanism*. The abelian version of the Higgs mechanism — the photon becomes massive when a charged scalar field acquires a non-zero vacuum expectation value — was discussed in class in the Fall semester, but please refresh your memory. I will discuss the non-abelian version during the last week of this semester, but I’d like you to read about it first so it wouldn’t come as a shock. Pay attention to the Georgi–Glashow model and make sure you understand why — and how — two of the three gauge bosons become massive while the third remains massless.

3. Finally, an exercise: Consider the pseudoscalar Yukawa theory. In a previous homework (set#18, problem 2), you (should) have calculated to one-loop order the infinite parts of all the counterterms in this theory. Let us now use the results of all this work to calculate the one-loop $\beta$–functions

$$ \frac{\partial g(M)}{\partial \log M} = \beta_g(g, \lambda) \quad \text{and} \quad \frac{\partial \lambda(M)}{\partial \log M} = \beta_\lambda(g, \lambda). \quad (1) $$

(a) Argue that for off-shell renormalization conditions at some renormalization scale $M \gg m_f, m_s$, each of the logarithmically divergent counterterms $\delta_\phi^Z$, $\delta_\psi^Z$, $\delta_g$ and $\delta_\lambda$ is

$$ \delta = C \log \left( \frac{\mu_{\text{cutoff}}^{\text{UV}}}{M^2} \right)^2 + \text{finite} \quad (2) $$

or

$$ \delta = C \left( \frac{1}{\epsilon} + \log \frac{\mu^2}{M^2} \right) + \text{finite} \quad (3) $$

in dimensional regularization, where the coefficient $C$ of the UV divergence has the same value for all renormalization schemes (on-shell or off-shell) and the ‘finite’ terms are not only finite but also $M$ independent.
(b) Show that the bare and the renormalized couplings of the Yukawa theory are related to each other as

$$\lambda + \delta\lambda = \lambda_0 Z^2_\phi, \quad g + \delta g = g_0 Z_\psi \sqrt{Z_\phi}, \quad (4)$$

and use these relations to derive

$$\beta_\lambda = 2(\lambda + \delta\lambda) \frac{\partial \log Z_\phi}{\partial \log M} - \frac{\partial \delta\lambda}{\partial \log M},$$
$$\beta_g = (g + \delta g) \left( \frac{\partial \log Z_\psi}{\partial \log M} + \frac{1}{2} \frac{\partial \log Z_\phi}{\partial \log M} \right) - \frac{\partial \delta g}{\partial \log M}. \quad (5)$$

At the one-loop level, these formulæ simplify to

$$\beta_\lambda = 2\lambda \frac{\partial \delta_\phi}{\partial \log M} - \frac{\partial \delta\lambda}{\partial \log M},$$
$$\beta_g = g \left( \frac{\partial \delta_\psi}{\partial \log M} + \frac{1}{2} \frac{\partial \delta_\phi}{\partial \log M} \right) - \frac{\partial \delta g}{\partial \log M}. \quad (6)$$

(c) Use the results of homework set #18 to evaluate the right hand sides of eqs. (6) and write down the $\beta$–functions (1) as functions of the renormalized couplings $g$ and $\lambda$. 