Problem 1(a): 
In 3-vector notations, the Lorentz algebra comprises the \( \hat{J} \) and \( \hat{K} \) operators satisfying the commutation relations

\[
[\hat{J}^i, \hat{J}^j] = i \varepsilon^{ijl} \hat{J}^l, \quad [\hat{J}^i, \hat{K}^j] = i \varepsilon^{ijl} \hat{K}^l, \quad [\hat{K}^i, \hat{K}^j] = -i \varepsilon^{ijl} \hat{J}^l.
\] (S.1)

Consequently, for the \( \hat{J}^\pm = \frac{1}{2} (\hat{J}^x \pm i \hat{K}^x) \), we have

\[
[\hat{J}^i_\pm, \hat{J}^j_\pm] = \frac{i}{4} \varepsilon^{ijl} \hat{J}^l \mp \frac{1}{4} \varepsilon^{ijl} \hat{K}^l \mp \frac{1}{4} \varepsilon^{ijl} \hat{J}^l = i \varepsilon^{ijl} \hat{J}^l = \varepsilon^{ijl} \hat{J}^l \pm (S.2)
\]

while

\[
[\hat{J}^i_\pm, \hat{J}^j_\mp] = \frac{i}{4} \varepsilon^{ijl} \hat{J}^l \mp \frac{1}{4} \varepsilon^{ijl} \hat{K}^l \pm \frac{1}{4} \varepsilon^{ijl} \hat{J}^l - \frac{i}{4} \varepsilon^{ijl} \hat{J}^l = 0. \quad (S.3)
\]

Q.E.D.

Problem 1(b): 
First, note the hermiticity of the \( \sigma^\mu \) matrices and the fact that any hermitian \( 2 \times 2 \) matrix is a unique linear combination of the four \( \sigma^\nu \) with real coefficients. Consequently,

\[
\forall M : \ M \sigma^\mu M^\dagger = \sigma^\nu L^\mu_\nu(M) \implies X'_\nu = L^\mu_\nu(M)X^\mu \quad (S.4)
\]

for some real \( 4 \times 4 \) matrix \( L^\mu_\nu(M) \). Furthermore, for \( M \in SL(2, \mathbb{C}) \), i.e. for \( \det(M) = 1 \), this \( L^\mu_\nu(M) \) matrix defines a Lorentz transform for which \( X'_\mu X'^\mu = X^\mu X^\mu \). To see this, we note that

\[
\det(X_\mu \sigma^\mu) = \det\begin{pmatrix} X_0 + X_3 & X_1 - iX_2 \\ X_1 + iX_2 & X_0 - X_3 \end{pmatrix} = (X_0)^2 - (X_3)^2 - (X_1)^2 - (X_2)^2 \equiv X^2 \quad (S.5)
\]

and then calculate

\[
X'^2 = \det(X'_\mu \sigma^\mu) = \det(M(X_\mu \sigma^\mu)M^\dagger) = |\det(M)|^2 \times \det(X_\mu \sigma^\mu) = 1 \times X^2. \quad (S.6)
\]

Also, the Lorentz transform \( X_\mu \rightarrow X'_\mu = L^\nu_\mu X_\nu \) is orthochronous because

\[
L^0_0 = \frac{1}{2} \text{tr}(\sigma^\nu L^0_\nu) = \frac{1}{2} \text{tr}(M \sigma^0 M^\dagger) = \frac{1}{2} \text{tr}(MM^\dagger) > 0. \quad (S.7)
\]
Problem 1(b*):
The simplest proof the $L^{\nu}_{\mu}(M)$ is proper as well as orthochronous involves the group law (problem 2(c) below) and the explicit examples of a pure rotation and a pure boost (problem 2(d) below, eqs. (S.11) and (S.13)), both of which are manifestly proper.

For any $SL(2,\mathbb{C})$ matrix $M$ we may decompose $M = HU$ where $H = \sqrt{MM^\dagger}$ is hermitian and $U = H^{-1}M$ is unitary. (Proof: $UU^\dagger = H^{-1}MM^\dagger H^{-1} = H^{-1}H^2H^{-1} = 1$.) Furthermore, both $H$ and $U$ are unimodular ($\det(H) = \det(U) = 1$), or in other words $H, U \in SL(2,\mathbb{C})$, which allows us to define two separate Lorentz transforms $L(H)$ and $L(U)$. According to the group law, together these two transform accomplish the $L(M)$ transform,

$$L(M) = L(H) \times L(U).$$

(S.8)

Now, $H$ is hermitian, unimodular, and positive definite, hence it has a well-defined logarithm which is hermitian and traceless, $\text{tr}(\log H) = 0$. For the $2 \times 2$ matrices, this means $\log H = -\frac{1}{2}r \sigma$ for some real 3-vector $r$, or equivalently $H = \exp(-\frac{1}{2}r \sigma)$. As we shall see in eq. (S.13) below, this means that $L(H)$ is a pure Lorentz boost of rapidity $r$ in the direction $n$. This boost manifestly does not invert space or time, thus $L(U)$ is proper.

Likewise, $U$ is unitary and unimodular, thus $U \in SU(2)$ and defines a pure rotation of space. Indeed, any $U \in SU(2)$ can be written as $U = \exp(-\frac{i}{2} \theta n' \sigma)$ for some angle $\theta$ and some axis $n'$, and according to eq. (S.11) below $L(U)$ is indeed a pure space rotation by angle $\theta$ around axis $n'$. Again, this rotation is proper — it does not invert space or time. Thus, $L(H)$ and $L(U)$ are both proper Lorentz transforms, hence their product $L(M)$ must also be proper. (Proof: $\det(L(M)) = \det(L(H)) \times \det(L(U)) = +1$.) Q.E.D.

And by the way, since any proper, orthochronous Lorentz transform $L \in SO^+(1,3)$ can be realized as $L(M)$ for some $M \in SL(2,\mathbb{C})$, it follows that any such transform is a product of a pure space rotation $L(H)$ followed by a pure Lorentz boost $L(U)$.

Problem 1(c):

$$\sigma_\lambda L^\lambda_{\mu}(M_1 M_2) = (M_2 M_1) \sigma_\mu(L^\mu_{\nu}(M_2 M_1)^\dagger = M_2 \left(M_1 \sigma_{\mu} M_1^\dagger = \sigma_{\nu} L^\nu_{\mu}(M_1)\right) M_2^\dagger$$

(S.9)

and hence $L^\lambda_{\mu}(M_2 M_1) = L^\lambda_{\nu}(M_2) L^\nu_{\mu}(M_1)$, i.e. $L(M_2 M_1) = L(M_2) L(M_1)$. Q.E.D.
Problem 1(d):

Let \( M = \exp\left(-\frac{i}{2} \theta \mathbf{n} \sigma\right) = \cos \frac{\theta}{2} - i \sin \frac{\theta}{2} \mathbf{n} \sigma \) and hence \( M^\dagger = M^{-1} = \cos \frac{\theta}{2} + i \sin \frac{\theta}{2} \mathbf{n} \sigma \). Given \( \sigma^0 = 1 \) and unitarity of \( M \), we have \( M \sigma^0 M^\dagger = \sigma^0 \), and hence according to eq. (7) \( t' = t \) regardless of \( x \). In other words, \( L(M) \) does not affect the time and is a purely spatial rotation. Specifically,

\[
\sigma \cdot x' = M(x \sigma) M^\dagger = \cos \frac{\theta}{2} (x \sigma) - i \sin \frac{\theta}{2} \sigma \left\{ n \sigma , (t - x) \sigma \right\} = 2i(n \times x) \cdot \sigma
\]

\[
+ \sin \frac{\theta}{2} \left( \left( n \sigma \right)(x \sigma)(n \sigma) - 2(nx)(n \sigma) - (x \sigma) \right)
\]

\[
= \cos \theta (x \sigma) + \sin \theta ((n \times x) \sigma) + (1 - \cos \theta)(nx)(n \sigma),
\]

thus

\[
x' = \cos \theta (x - n(nx)) + \sin \theta n \times x + n(nx)
\]

which indeed describes a rotation through angle \( \theta \) around axis \( n \).

Now consider \( M = \exp\left(-\frac{r}{2} \mathbf{n} \sigma\right) = \cosh \frac{r}{2} - \sinh \frac{r}{2} \mathbf{n} \sigma \) and hence \( M^\dagger = M \). In this case, we have

\[
M \left( x^\mu \sigma_\mu \equiv t - x \sigma \right) M^\dagger = \cosh \frac{r}{2} (t - x \sigma)
\]

\[
- \sinh \frac{r}{2} \cosh \frac{r}{2} \left\{ n \sigma , (t - x) \sigma \right\} = 2t(n \sigma) - 2(nx)
\]

\[
+ \sinh^2 \frac{r}{2} \left( \left( n \sigma \right)(t - x \sigma)(n \sigma) = t - 2(nx)(n \sigma) + (x \sigma) \right)
\]

\[
= (\cosh r t + \sinh r nx) - (\sigma n) (\sinh r t + \cosh r nx) - \sigma \cdot (x - n(nx)),
\]

and therefore,

\[
t' = (\cosh r) t + (\sinh r) nx, \quad x' = n((\sinh r) t + (\cosh r) nx) + (x - n(nx))
\]

which is precisely the Lorentz boost of rapidity \( r \) in the direction \( n \). (The rapidity \( r \) is related to the usual parameters of a Lorentz boost according to \( \beta = \tanh r, \gamma = \cosh r, \gamma \beta = \sinh r \). For several boosts in the same directions, the rapidities add up, \( r_{\text{tot}} = r_1 + r_2 + \cdots \).)

Q.E.D.

Problem 1 (e):

For any Lie algebra equivalent to an angular momentum or its analytic continuation, the product
of two doublets comprises a triplet and a singlet, \(2 \otimes 2 = 3 \oplus 1\), or in \((j)\) notations, \((\frac{1}{2}) \otimes (\frac{1}{2}) = (1) \oplus (0)\). Furthermore, the triplet \(3 = (1)\) is symmetric with respect to permutations of the two doublets while the singlet \(1 = (0)\) is antisymmetric.

For two separate and independent types of angular momenta \(J_+\) and \(J_-\) we combine the \(j_+\) quantum numbers independently of \(j_-\) and the \(j_-\) quantum numbers independently of \(j_+\). Thus,

\[
(\frac{1}{2}, \frac{1}{2}) \otimes (\frac{1}{2}, \frac{1}{2}) = (1, 1) \oplus (1, 0) \oplus (0, 1) \oplus (0, 0).
\]

Furthermore, the symmetric part of this product should be either symmetric with respect to both the \(j_+\) and the \(j_-\) indices or antisymmetric with respect to both indices, thus

\[
[(\frac{1}{2}, \frac{1}{2}) \otimes (\frac{1}{2}, \frac{1}{2})]_{\text{sym}} = (1, 1) \oplus (0, 0).
\]

Likewise, the antisymmetric part is either symmetric with respect to the \(j_+\) but antisymmetric with respect to the \(j_-\) or the other way around, thus

\[
[(\frac{1}{2}, \frac{1}{2}) \otimes (\frac{1}{2}, \frac{1}{2})]_{\text{antisym}} = (1, 0) \oplus (0, 1).
\]

From the \(SO(1,3)\) point of view, the \((\frac{1}{2}, \frac{1}{2})\) multiplet is the Lorentz vector, hence the generic 2–index Lorentz tensor decomposes into irreducible multiplets according to eq. (S.14). Imposing symmetry conditions, we have eq. (S.15) for the symmetric 2–index tensor \(T^{\mu \nu} = T^{\nu \mu}\) where the singlet \((0, 0)\) corresponds to the trace \(T^\mu_\mu\) while the \((1, 1)\) irreducible multiplet is the traceless symmetric tensor.

Likewise, the antisymmetric Lorentz tensor \(F^{\mu \nu} = -\ F^{\nu \mu}\) decomposes according to eq. (S.16). Here, the irreducible components \((1, 0)\) and \((0, 1)\) are complex but conjugate to each other; individually, they describe antisymmetric tensors subject to complex duality conditions \(\frac{1}{2} \epsilon^{\kappa \lambda \mu \nu} F_{\mu \nu} = \pm i F^{\kappa \lambda}\), i.e. \(E = \pm i B\).

Problem 1(f):
Without the \(\gamma_\mu \Psi^\mu = 0\) constraint, the spin-vector \(\Psi^\mu_a\) is the tensor product or the Dirac spinor and the Lorentz vector, thus

\[
\left[(\frac{1}{2}, 0) \oplus (0, \frac{1}{2})\right] \otimes (\frac{1}{2}, \frac{1}{2}) = (1, \frac{1}{2}) \oplus (0, \frac{1}{2}) \oplus (\frac{1}{2}, 1) \oplus (\frac{1}{2}, 0).
\]

The constraint removes a Dirac spinor \(\gamma_\mu \Psi^\mu \Rightarrow (\frac{1}{2}, 0) \oplus (0, \frac{1}{2})\), thus we are left with the \((1, \frac{1}{2}) \oplus (\frac{1}{2}, 1)\) part for the Rarita–Schwinger spin-vector.
Problem 2(a):

In spacetime, any purely spatial rotation $R(t, x) = (t, Rx)$ commutes with the space reflection $P(t, x) = (t, -x)$. Consequently, by the group law, the parity operator $\hat{P}$ in the Fock space must commute with operators $\hat{D}(R)$ representing the space rotations. And since the spatial rotations are generated by the angular momentum components $\hat{J}^i$, $\hat{D}(\theta, n) = \exp(-i\theta n\hat{J})$, the fact that all the $\hat{D}(\theta, n)$ commute with the parity operator implies that $\hat{J}\hat{P} = \hat{P}\hat{J}$.

Next, consider a pure Lorentz boost $B(r, n)$ (S.13). Clearly, reflecting the space reverses the direction of the boost,

$$PB(r, n)P = B(r, -n) = B^{-1}(r, n), \quad (S.18)$$

hence the operators

$$\hat{D}(r, n) = \exp(-irn\hat{K}) \quad (S.19)$$

representing the pure Lorentz boosts in the Fock space must have similar commutation relations with the parity operator:

$$\hat{P}\hat{D}(r, n)\hat{P} = \hat{D}(r, -n). \quad (S.20)$$

Consequently, in terms of the boost generators $\hat{K}^i$,

$$\hat{P}\exp(-irn\hat{K})\hat{P} = \exp(+irn\hat{K}) \implies \hat{P}\hat{K}\hat{P} = -\hat{K}. \quad (S.21)$$

And in terms of the $\hat{J}_\pm$ operators (1), the fact that $\hat{P}$ commutes with $\hat{J}$ but anticommutes with $\hat{K}$ means

$$\hat{P}\hat{J}_+\hat{P} = \hat{J}_- \quad \text{and} \quad \hat{P}\hat{J}_-\hat{P} = \hat{J}_+; \quad (S.22)$$

in other words, parity interchanges the $\hat{J}_+$ and the $\hat{J}_-$ operators.

Now consider a multiplet $\hat{\varphi}_a(x)$ of quantum fields. Saying that this multiplets has definite values of $j_+ = C$ and $j_- = D$ technically means that

$$\left[\hat{J}_+^2, \hat{\varphi}_a(x)\right] = C(C + 1)\hat{\varphi}_a(x),$$

$$\left[\hat{J}_-^2, \hat{\varphi}_a(x)\right] = D(D + 1)\hat{\varphi}_a(x). \quad (S.23)$$
Applying the parity operator to the $\varphi_a(x)$ fields, we have

$$\hat{\varphi}_a'(t,-x) = \hat{P}\hat{\varphi}_a(t,x)\hat{P}. \quad (S.24)$$

Consequently,

$$\begin{align*}
\left[ \hat{J}^2_+, \hat{\varphi}_a'(t,-x) \right] & = \hat{P}\left[ \hat{P}\hat{J}^2_+\hat{P}, \hat{P}\hat{\varphi}_a'(t,-x)\hat{P} \right] \hat{P} \quad \langle \text{note } \hat{P}^2 = 1 \rangle \\
& = \hat{P}\left[ \hat{J}^2_+, \hat{\varphi}_a(t,x) \right] \hat{P} \quad \langle \text{by eqs. (S.22) and (S.24)} \rangle \\
& = \hat{P}\left( D(D+1)\hat{\varphi}_a(t,x) \right) \hat{P} \quad \langle \text{by eq. (S.23)} \rangle \\
& = D(D+1)\hat{\varphi}_a'(t,-x),
\end{align*}$$

and likewise

$$\begin{align*}
\left[ \hat{J}^2_+, \hat{\varphi}_a'(t,-x) \right] & = C(C+1)\hat{\varphi}_a'(t,-x). \quad (S.25)
\end{align*}$$

Thus, the field multiplet $\hat{\varphi}_a'(x')$ has definite $(j_+,j_-)' = (D,C)$, which exchanges the $j_+$ and the $j_-$ quantum numbers $(j_+,j_-) = (C,D)$ of the original multiplet $\hat{\varphi}_a(x)$.

**Problem 2(b):**

First, consider Dirac equation. Rewriting eq. (10) as $\Psi'(x',t') = \pm \gamma^0\Psi(x = -x', t = +t')$, we have

$$\begin{align*}
(i \not\partial - m)\Psi'(x') & \equiv (i\gamma^0\partial_0 + i\vec{\gamma} \cdot \nabla' - m) \times (\pm \gamma^0)\Psi(x',t) \\
& = (\pm \gamma^0)(i\gamma^0\partial_0 - i\vec{\gamma} \cdot \nabla' - m)\Psi(x',t) \\
& = (\pm \gamma^0)(i\gamma^0\partial_0 + i\vec{\gamma} \cdot \nabla - m)\Psi(-x,t) \\
& \equiv (\pm \gamma^0)(i \not\partial - m)\Psi \bigg|_{x'}. \quad (S.27)
\end{align*}$$

Thus, $(i \not\partial - m)\Psi(x)$ transforms under parity exactly as the Dirac field $\Psi(x)$ itself, which means that the Dirac equation is **covariant** under parity.

Now consider Dirac Lagrangian $\mathcal{L}\bar{\Psi}(i \not\partial - m)\Psi$. Conjugating eq. (10) we have $\bar{\Psi}(x',t') = \pm \bar{\Psi}(x = -x', t = +t')$, and hence in light of eq. (S.27),

$$\begin{align*}
\mathcal{L}'(x') & = \bar{\Psi}'(i \not\partial' - m)\Psi'(x') = \overline{\bar{\Psi}(i \not\partial - m)\Psi}(x) = \mathcal{L}(x). \quad (S.28)
\end{align*}$$

In other words, Dirac Lagrangian transforms under parity as a true scalar field, and consequently the Dirac Action $\int d^4x \mathcal{L}$ is invariant.
Problem 3(a):
As explained in class,
\[ u(p, s) = \left( \frac{\sqrt{E - p\sigma}}{\sqrt{E + p\sigma}}, \xi_s^\dagger \sqrt{E - p\sigma} \right) \Rightarrow \bar{u}(p, s) = \left( \xi_s^\dagger \sqrt{E + p\sigma}, \xi_s^\dagger \sqrt{E - p\sigma} \right) \]  
(S.29)

where \( \xi_s \) is the ordinary 3D 2–component spinor normalized to \( \xi_s^\dagger \xi = 1 \) and therefore
\[ \sum_s (\xi_s \xi_s^\dagger) = 1 \]  
(S.30)
as a \( 2 \times 2 \) matrix. Consequently, in \( 4 \times 4 \) matrix notations, we have
\[ \sum_s u(p, s)\bar{u}(p, s) = \sum_s \begin{pmatrix} \sqrt{E - p\sigma} (\xi_s \xi_s^\dagger) \sqrt{E + p\sigma} & \sqrt{E - p\sigma} (\xi_s \xi_s^\dagger) \sqrt{E - p\sigma} \\ \sqrt{E + p\sigma} (\xi_s \xi_s^\dagger) \sqrt{E + p\sigma} & \sqrt{E + p\sigma} (\xi_s \xi_s^\dagger) \sqrt{E - p\sigma} \end{pmatrix} = m + \phi. \]  
(S.31)

Likewise, for the negative-frequency spinors we have
\[ v(p, s) = \begin{pmatrix} +\sqrt{E - p\sigma} \eta_s \\ -\sqrt{E + p\sigma} \eta_s \end{pmatrix} \Rightarrow \bar{v}(p, s) = \begin{pmatrix} -\eta_s^\dagger \sqrt{E + p\sigma} \\ \eta_s^\dagger \sqrt{E - p\sigma} \end{pmatrix} \]  
(S.32)
and therefore
\[ \sum_s v(p, s)\bar{v}(p, s) = \sum_s \begin{pmatrix} -\sqrt{E - p\sigma} (\xi_s \xi_s^\dagger) \sqrt{E + p\sigma} & +\sqrt{E - p\sigma} (\xi_s \xi_s^\dagger) \sqrt{E - p\sigma} \\ +\sqrt{E + p\sigma} (\xi_s \xi_s^\dagger) \sqrt{E + p\sigma} & -\sqrt{E + p\sigma} (\xi_s \xi_s^\dagger) \sqrt{E - p\sigma} \end{pmatrix} = \begin{pmatrix} m \\ m \end{pmatrix} = -m + \phi. \]  
(S.33)
Problem 3(b):
The constant spinors \( u \equiv u(p, s) \) and \( \bar{u}' \equiv \bar{u}(p', s') \) satisfy Dirac equations \( \not{p} u = m u \) and \( \not{p}' \bar{u}' = m \bar{u}' \). Applying both equations to the Dirac “sandwich” \( \bar{u}' \gamma^\mu u \), we have
\[
\bar{u}' \gamma^\mu u = \frac{1}{m} \bar{u}' \not{p}' \times \gamma^\mu u = \frac{1}{m} \bar{u}' \gamma^\mu \not{p} u = \frac{1}{2m} \bar{u}' (\not{p} \gamma^\mu + \gamma^\mu \not{p}) u. \tag{S.34}
\]
Furthermore,
\[
\not{p}' \gamma^\mu + \gamma^\mu \not{p} \equiv \not{p}' \nu \gamma^\nu \gamma^\mu + p_\nu \gamma^\mu \gamma^\nu = \frac{1}{2} (p' + p)_\nu [\gamma^\mu, \gamma^\nu] + \frac{1}{2} (p' - p)_\nu \gamma^\mu \gamma^\nu
\]
and therefore
\[
\bar{u}' \gamma^\mu u = \frac{(p' + p)_\mu}{2m} \bar{u}' u + \frac{i(p' - p)_\mu}{m} \bar{u}' S^\mu\nu u. \tag{2}
\]
Q.E.D.

Problem 3(c):
The negative-frequency spinors \( v \equiv v(p, s) \) and \( \bar{v}' \equiv \bar{v}(p', s') \) satisfy Dirac equations \( \not{p} v = -m v \) and \( \not{p}' \bar{v}' = -m \bar{v}' \). Consequently, proceeding exactly as above modulo signs, we have
\[
\bar{u}' \gamma^\mu v = \frac{(p' - p)_\mu}{2m} \bar{u}' v + \frac{i(p' + p)_\mu}{m} \bar{u}' S^\mu\nu v, \\
\bar{v}' \gamma^\mu u = \frac{(-p' + p)_\mu}{2m} \bar{v}' u + \frac{i(-p' + p)_\mu}{m} \bar{v}' S^\mu\nu u, \tag{S.36}
\\
\bar{v}' \gamma^\mu v = \frac{(-p' - p)_\mu}{2m} \bar{v}' v + \frac{i(-p' + p)_\mu}{m} \bar{v}' S^\mu\nu v.
\]

Problem 3(d):
Given \( p' = -p \) and hence \( E' = E = \pm \sqrt{p^2 + m^2} \), we have
\[
v(p', s') = \begin{pmatrix} +\sqrt{E' - \not{p} \sigma \eta} \\ -\sqrt{E' + \not{p} \sigma \eta} \end{pmatrix} = \begin{pmatrix} +\sqrt{E + \not{p} \sigma \eta} \\ -\sqrt{E - \not{p} \sigma \eta} \end{pmatrix}
\]
where \( \eta = \eta_{s'} \). At the same time,
\[
u^\dagger(p, s) = \begin{pmatrix} \xi^\dagger \sqrt{E - \not{p} \sigma} \\ \xi^\dagger \sqrt{E + \not{p} \sigma} \end{pmatrix}
\]
\[\text{where } \eta = \eta_{s'}. \text{ At the same time,}
\[
\nu^\dagger(p, s) = \begin{pmatrix} \xi^\dagger \sqrt{E - \not{p} \sigma} \\ \xi^\dagger \sqrt{E + \not{p} \sigma} \end{pmatrix}
\]
\[\text{where } \eta = \eta_{s'}. \text{ At the same time,}
\[
\nu^\dagger(p, s) = \begin{pmatrix} \xi^\dagger \sqrt{E - \not{p} \sigma} \\ \xi^\dagger \sqrt{E + \not{p} \sigma} \end{pmatrix}
\]
where \( \xi = \xi_s \). Consequently,

\[
   u^\dagger(p, s)v(p', s') = \xi^\dagger \sqrt{E - p\sigma} \times \sqrt{E + p\sigma} \eta - \xi^\dagger \sqrt{E + p\sigma} \times \sqrt{E - p\sigma} \eta = 0, \quad Q.E.D.
\]