1. First, finish the textbook problem 9.2 — solve parts (d) and (e). Also, generalize part (d) to free fermionic fields in $3 + 1$ dimensions.

2. Next, consider a scalar analogue of QCD, or more generally a theory of Yang–Mills fields $A^a_\mu$ and complex scalars $\Phi_i$ in some multiplet $(r)$ of the gauge group $G$.

   (a) Write down the Lagrangian and the Feynman rules of this theory.

Next, consider the annihilation process $\Phi + \Phi^* \rightarrow 2$ gauge bosons. At the tree level, there are four Feynman diagrams contributing to this process.

   (b) Draw the diagrams and write down the tree-level annihilation amplitude.

As discussed in class, amplitudes involving the non-abelian gauge fields satisfy a weak form of the Ward identity: On-shell Amplitudes involving a longitudinally polarized gauge bosons vanish, provided all the other gauge bosons are transversely polarized. In other words,

$$M \equiv e^{\mu_1}_{a_1} e^{\mu_2}_{a_2} \cdots e^{\mu_n}_{a_n} M_{\mu_1, \mu_2, \cdots, \mu_n}(\text{momenta}) = 0$$

when $e^{\mu}_{a_1} \propto k^{\mu}_{a_1}$ but $e^{\nu}_{a} k^{\nu}_{a} = \cdots = e^{\nu}_{n} k^{\nu}_{n} = 0$.

   (c) Verify this identity for the scalar annihilation amplitude.

3. Finally, a big reading assignment: §12.1 of the Peskin & Schroeder textbook about integrating out of the short-distance modes and the Wilsonian renormalization. Also, read §12.4 of the Weinberg’s book (vol. 1) about the same subjects.

These issues are important, and I wish I could spend a lecture or two explaining them. Alas, the class time is too short, hence this assignment.